

## ERME column

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ERME Thematic Working Groups. *The European Society for Research in Mathematics Education (ERME) holds a biennial conference (CERME), in which research is presented and discussed in Thematic Working Groups (TWG). We continue the initiative of introducing the working groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education could be interesting or relevant for research mathematicians. Our aim is to extend the ERME community to new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.*

### Introducing CERME Thematic Working Group 2 – Arithmetic and Number Systems

Learning arithmetic and number systems is a relevant field in mathematics education. Kindergarten students make their first steps in developing a number concept of natural numbers with counting and subitizing. Primary school aims to develop a deep conceptual understanding of numbers and basic arithmetic operations, and, a main focus is on building number- and structure-sense as foundation for flexibility in mental arithmetic. A deep understanding of natural numbers is an important prerequisite for developing conceptual understanding and procedural skills in rational numbers in secondary school. Learning arithmetic and number systems is not only a matter of content. Beyond the content, arithmetic education research focuses on models for teaching and learning arithmetic, approaches for heterogeneous and inclusive classrooms, analogue and digital tools to support understanding, and, let us not forget, cultural practices.

Teaching and learning arithmetic and number systems through activities in kindergarten and school is a broad field. For that reason, the thematic working group TWG2 as a forum for presenting and discussing research and development in this subject area is characterized by a great variety of topics. The scope of the TWG comprises kindergarten to 12th grade, and emphasizes for instance research-based specifications of domain-specific goals, analysis of learning processes and learning outcomes in domain-specific learning environments and classroom cultures, as well as new approaches to the design of meaningful and rich learning environments and assessments.

*Brief History of TWG2.* TWG2 is a relatively young working group which started at CERME7 (2011) in Rzeszów (Poland) with a small group with nine papers, which nevertheless covered various topics including place value, number concepts, adaptive and flexible use of strategies, number sense, fractions in primary education, qualitative and quantitative reasoning, representations and collaborative learning. Two years later, at CERME8 in Antalya (Turkey), the group discussed thirteen papers and one poster. The papers dealt with topics related to natural numbers, fractions and decimals, proportion and negative numbers, and mainly investigated learners' thinking and understanding related to particular mathematics content. Developing conceptual understanding and flexibility in primary and secondary students was shared common ground. The TWG grew steadily, and at CERME9 in Prague (Czech Republic), twenty-one papers and one poster were submitted. The various topics can be categorized in three thematic groups:

- (1) number sense, conceptual understanding and flexibility;
- (2) the role of models;
- (3) subject matter analysis.

Even if the number of discussed papers and posters slightly decreased, CERME10 was characterized by an even greater variety of themes. Fifteen papers and one poster addressing research for different ages and different approaches were intensively discussed. The key themes were number sense and structure sense, estimation and estimation tasks, flexibility in mental calculation, derived fact-based strategies for multiplication in low-achieving students, understanding of rational numbers and ratios, didactic models as scaffolds for the evolution of mathematical knowledge, as well as teachers' knowledge about rational numbers, ratio and place value. The most recent meeting of TWG2 was CERME11. While CERME9 and CERME10 were dominated by design-based research, CERME11 was characterized by a balance of qualitative and quantitative approaches (more details below).

*Working spirit of TWG2.* TWG2 is a diverse group which gathers researchers from different domains (mathematics education, psychology, mathematics), and from different levels of experience. Approximately one third of the researchers have been with the

group from the beginning, and have built a stable basis for developing common ideas and concepts. The fluctuation of group members between the conferences is challenging in terms of developing a common understanding of concepts, but it is also a great opportunity for enhancing our perspectives and notions.

The TWG work always starts with a small working-group with two aims: firstly, to give all participants – especially researchers who are new to the group – an opportunity to engage in personal interaction, and secondly, to engage everybody in an intensive discussion from the start, especially those people who are not as fluent and secure in the conference language as others. According to our experience of recent years, this small group discussion on thematically related papers provides a fruitful basis for joint work during the conference week. This is especially efficient because we assign each group member to two related papers four weeks in advance, and provide specific questions for individual preparation.

CERME has the spirit of a working conference, and participants are supposed to read all the papers of the TWG in advance. This makes it possible to place more emphasis on discussion and scientific exchange during the joint work sessions. Depending on the amount of submitted papers, it is our goal to discuss each paper at least for 30 minutes after a short introduction in the form of a five-minute-reminder. Finally, each group member gives written feedback after each discussion.

*Current and further work of TWG2.* Due to the specific situation caused by the Covid-19-pandemic, the last time TWG2 was able to meet was in spring 2019 at CERME 11 in Utrecht (Netherlands). Here, twenty papers and four posters that reflected the richness of the field were introduced and discussed. The group had the chance to broaden the discussion by regarding the same topics from the perspectives of both cognitive psychologists and mathematics education researchers. In comparison to the TWG work of the previous years, there was also a change regarding the approach to research.

The various papers presented and discussed were grouped into four overarching themes: The role of manipulatives in learning processes in early arithmetic, learning and teaching numbers and operations in kindergarten to first grade, learning and teaching arithmetic in second to sixth grade, and various approaches to learning rational numbers. Additionally, there was one paper on metacognition. Like all the previous years, the group work is characterized by a great variety of topics and approaches. This is always a challenge for developing a common understanding, but it is also the strength of TWG2. The great variety of papers under discussion gave rise to interesting and often animated discussions that went far beyond the specific topics of single papers. We worked out differences and commonalities of used terms, concepts, theoretical frameworks and methodological approaches. Once again, we identified different conventions of naming numbers and operations, different understandings of terms and concepts such as number

sense, flexibility and mastery, and partly conflicting views of quantitative and qualitative approaches. Altogether, we broadened our own culturally-influenced perspectives and drew new inspiration for further research.

The aspects we discussed were manifold and reflected the whole spectrum of learning arithmetic in primary and secondary level, dealing with such topics as:

- The role and relevance of counting in different cultures and curricula, and a critical reflection on counting for developing strategies in mental calculation.
- The importance of designing learning environments based on our knowledge of students' strategies, conceptions and misconceptions.
- The emphasis on fostering students' conceptual knowledge regarding numbers and operations in different number systems.
- The clarification of methodology regarding what is measured, why measures are used, what is meant by a result, and how it contributes to a better understanding of teaching and learning.
- The challenges of transition between number systems, between representations, and from intervention research to students' long-term understanding.

Our research is characterized by a huge variety of questions, yet it is driven by a common goal: supporting understanding and meaningful learning in all students so as to develop their conceptual knowledge, number sense and structure sense in different number systems, as well as flexible and adaptive expertise in arithmetic and number systems. Empirical results suggest the early development of a conceptual understanding of numbers and basic arithmetic operations as a precursor for mathematical achievement in school (Krajewski & Schneider 2009). Learning processes, teaching approaches and learning trajectories from early arithmetic to university mathematics is an important field of research which opens up the possibility of joint work between mathematicians and math educators.

In Utrecht, we decided to meet in May 2020 for an ERME Topic Conference at the University of Leeds, to specify terms and work on common understanding of number sense, flexibility and mastery. All planning was successfully completed in February 2020, and 32 abstracts were submitted and accepted, but we were then obliged to postpone the conference due to Covid-19. The situation still has not changed, and so the ETC will take place as a virtual conference on May, 12, 2021, with a young researcher day on May, 11, 2021 to enable joint work to take place.

## Reference

- [1] K. Krajewski and W. Schneider, Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learn. Instr.* 19, 513–526 (2009)

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Renata Carvalho holds a PhD in mathematics education. She collaborates with the UIDEF of the Institute of Education of the University of Lisbon as a researcher and with the School of Education of Lisbon in preservice education. She also works with in-service teachers in professional development courses. Her first participation in the TWG2 was in CERME8 as a young researcher, and has acted as co-leader since CERME10. Her research focus is mental computation and rational numbers at the elementary levels.  
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Judy Sayers is a lecturer and researcher at the University of Leeds in England. She first joined CERME5 in Cyprus, but joined the newly formed TWG2 Arithmetic group in CERME7 in Rzeszow where she continues to enjoy great conversation, debates and empowering new ideas. Her research interest in early years' mathematics led her to the development of a large funded project Foundational Number Sense (FoNS) with colleagues at Stockholm University, where she continues to collaborate.  
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