A conversation with Reuben Hersh

Ulf Persson

In the spring of 2009 the NCM (Nationellt Centrum för Matematik) a didactic institute located in Gotheburg and founded by Bengt Johansson, invited Marcus du Sautoy to give a popular lecture. I was invited along to the luncheon and dinner and was asked to interview the guest, but he was too busy, so instead I 'made up' an interview on the basis of his lecture, the conversations at meals, the discussion at the center, and snatches of interchanges during walks between venues. Thus I wrote down both questions and answers and submitted it to du Sautoy, who made some minor changes. The whole interview was published in the Newsletter of the Swedish Math Society, for which I was conveniently the chief editor. The director Johansson liked it so much that he had the whole procedure repeated with Keith Devlin later that summer and the next year he wanted to invite Ian Stewart, who could not come, so instead I was dispatched to Warwick University. The following winter I was sent to Boston, where I had a few sessions with my old advisor David Mumford, and then I went to New Mexico where I spent a few days with Reuben Hersh. Later on I was sent to Paris talking to Cédric Villani, Yves Meyer and Luc Illusie. For those opportunities I am indebted to the generosity of the NCM, which is somewhat ironic as I have been a vocal critic of mathematical didacticians and their various claims to be scientific.

The routine I initiated with Sautoy has not only served me on the missions launched by NCM but also on other occasions, as when interviewing Fields Medalists at the ICM, the results of which have been published in the EMS. My inspiration has been Eckermann's *Gespräche mit Goethe* (Conversations with Goethe) and the procedure is straightforward. I have a discussion with the subject, ideally over several days, but that is not an option at an ICM, and I keep no notes, I make no recordings of anything but trust my memory. After the interview I may jot down some cryptic notes to myself as a support and then I sit down and make up a conversation, or if you prefer an interview. This is great fun as it is in the spirit of writing a play, for one thing you get to formulate both questions and answers, and the purpose is to give the illusion of a conversation which, at least in a literal sense, has never taken place. Some people may be shocked at this confession and dismiss it all as counterfeit; on the other hand the subject has the last word and can make as many amendments as they want (some

Ulf Persson, July 2021

subjects, such as Mumford and Hersh, got into the spirit of the thing and made extensive elaborations, but most have been satisfied with minor revisions). The conversation as such may have never taken place, nor have the involved used the exact formulations presented, but so what? As long as the subjects give their consents and blessings everything is fine. The form of an interview or, as I prefer, conversation, is just a way of conveying information and hopefully also the character of the people involved, and as such presents a lively way of so doing.

The earlier 'interviews' were published in The Newsletter of the Swedish Math Society and it was my intention to collect them all in a book but that ambition has not yet come to fruition. In the meantime nothing prevents me to let them appear to a larger audience and below I present the interview with Reuben Hersh which was conducted in his home in Santa Fe in February 2011. It was published in Bulletin of the Swedish Math Society in 2015 to which Hersh made some minor revisions in light of the time which had passed. Early in 2020 Hersh died at the age of 92 and the interview was also published on his memorial page through his son Daniel Hersh. My stay with Reuben and his partner Vera was very

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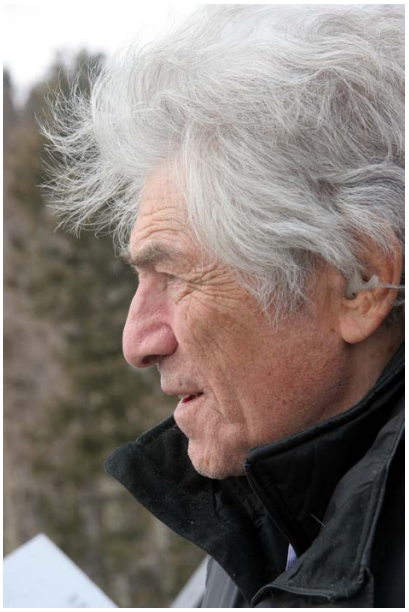
memorable and I regret that it turned out to be the only occasion I was going to have to meet him in the flesh; prior to this, we had kept up a rewarding e-mail exchange, which of course continued, and would do so until his death a decade later.

For an introduction to Reuben Hersh I refer to my recent obituary in the EMS Newsletter 116, June 2020. The original publication was in the Bulletin of the Swedish Math Society, October 2015, and we thank the society for permission to republish it.

Ulf Persson (UP): So you went to Harvard at fifteen, wasn't that very young?

Reuben Hersh (RH): There were a few sixteen-year olds as well. Smart Jewish kids from New York, who wouldn't have been let into Harvard in normal times. The war was on, this was 1943, and most of the regular Harvard boys were in the Navy, so we were let in to help fill the seats in the classrooms. Then after the war, things changed, it stopped being such a genteel snob school.



Reuben Hersh, February 2011

UP: Did you do any math at that time?

RH: I had enjoyed math a lot in high school. But my calculus course with David Widder was a disaster. Just plodding through motivationless technicalities. It killed all my interest. Instead I began to think of myself more as a writer. But Harvard didn't offer a degree in creative writing in those days. So instead I majored in English lit. I read a lot of fiction, poetry, drama. I wrote short lyric poems, and even won the Lloyd Mckim Garrison prize *for poetry by a Harvard undergraduate*. Two years running. (Not much competition, with most of the usual Harvard boys away in the Navy.) I still have the silver tombstone medal with my name on it, and I'm sure that the stuff that won the prize is still on file in some little room in Widener Library. My poetry professor Robert Hillyer nominated me to represent Harvard in an undergraduate poetry competition at Mt Holyoke, that's an Ivy League girl's college. I hitchhiked to Mt Holyoke, and showed up a bit late at the formal dinner. Wouldn't spend the money for a train ticket.

UP: So you gave up completely on math?

RH: I did have two friends,Johnny Wermer and Henry Helson, who later became successful mathematicians. They were a bit older than me. I just found out recently that Henry had died. That was sad. Have you heard of those guys? Wermer has a Swedish wife.

UP: Of course I have heard of them. So what did you do after you graduated?

RH: Well, I had assumed I was going to go into the Army to fight Hitler. As soon as I was 18, I could join the army without my parent's consent. But I was too late! The war ended in August 1945, and I wasn't 18 until December. I graduated in January 1946. But I had no other plans, no idea what else to do, and I expected to be drafted eventually any way, so I joined the army. They had 18-month enlistments available. I somehow survived basic training, and ended up classified as a clerk/typist. I spent seven months as a clerk typist in the U.S. Military Government headquarters in Seoul, Korea.

UP: What did you do next?

RH: I was discharged at Camp Stoneman in California and hitchhiked home to Mount Vernon, New York, my parent's home. I moved back in with them, and tried just sitting at home and writing, in order to become a writer …

UP: But it was not working, you were frustrated.

RH: Desperately. I just couldn't do it. None of it was any good. I was too young. I had no experiences worth writing about. Of course everyone has the experience of growing up – childhood and family life – but that has been written to death. I had nothing original to say.

UP: So you had to give up on that?

RH: Yes, I had to give it up. And I had to support myself, I had to get out of my parent's house. Somehow my mother knew someone who knew someone who was an editor on a magazine, and he knew Leon Svirsky, who was one of the editors of Scientific American. You know about Scientific American?

UP: Yes, of course. I remember they published retrospectives from 50 and 100 years earlier.

RH: That magazine had really deteriorated, it was almost dead, and three science writers from Time and Life magazine decided to buy it up, get the right to use the name, and do something different. This was 1947. Their idea was publish articles by scientists themselves, telling the public about their own work.

UP: It was a real discontinuity.

RH: Definitely. Of course most scientists can't write in a style accessible to the general public, but with a lot of help from editors, something actually readable may come out. They had just promoted the office boy, so there was an opening for me, working in the mail room. I spent a year packing up copies of Albert G. Ingalls' books on amateur telescope making. Then gradually I got more interesting things to do, minor writing and editing. I was assigned to read old issues and write those retrospectives you just referred to.

UP: But you did not want to spend the rest of your life there.

RH: I couldn't see myself doing this for the rest of my life. After four years there, I quit and tried to become a machinist. Being a veteran, I could take advantage of the GI Bill and get some paid schooling in the evenings, at the Machine and Metal Trades High School on York Avenue. Do you know what a lathe is?

UP: Of course I know.

RH: A lathe is a machine that cuts cylindrical parts. Machine parts that have to rotate are cylindrical. But manual work didn't come easy to me.

UP: You had trouble understanding the instructions.

RH: Right. I had a hard time understanding without clear instructions. They left so many obvious things out, things which were not obvious to me. But eventually I got reasonably good at it. Not great, but good enough to hold a job and make a living. You can learn a lot of things as long as you put your mind to it.

UP: But you did not spend your whole life there either.

RH: No, I did not. I had a very stupid accident using a band saw. I sawed off the upper half of my right thumb.

UP: And at the time there was no point to retrieve it and have it sewn on again. Micro-surgery had not yet been invented.

RH: It really scared me, and I decided I had to do something else. Everything I had done up to then – joining the army, trying to be a writer, trying to be a working class activist – it all was a way of trying to help change the world, fight Fascism and racism and oppression and so on. But it had all been a delusion, almost a waste of time. I had tried to change the world, and I just couldn't do it. Moreover, all this happened at almost the same time as Nikita Khrushchev's famous *secret speech* to the congress of the CPSU, where the head of the CPSU revealed that he and his associates had been servants of a paranoid sociopathic mass murderer. So finally I decided that if my efforts to change the world had been useless or worse, I might as well just do whatever I enjoyed. It took a while to figure out what I really enjoyed. Then I remembered that I used to enjoy mathematics and decided to apply to graduate school.

UP: And so you got into mathematics? But you had had very little mathematics, why should you be accepted as a graduate student?

RH: Fortunately, I applied at NYU, which had a somewhat open mind about off-beat applicants. I wanted to stay in New York, and between NYU and Columbia, NYU was a better bet. I was interviewed by a guy I had never heard of, a professor named Fritz John. Of course, later I knew very well who he was. He was skeptical, so I told him that I had gotten a perfect score on the math part of the Graduate Record Exam. He answered, *And what is the graduate record exam?* I explained that to him, and he thought about it, and said, *Probably means something*. So he told me to take advanced calculus in summer school, and then if I did OK I could apply for admission as a graduate student in the fall. Summer school was where I met Harold Shapiro. He had a summer job at NYU teaching advanced calculus. He is another mathematician who has a Swedish wife.

UP: You were quite old by then.

RH: Twenty-nine. A rather mature age to start in mathematics.

UP: But it had advantages?

RH: Unlike many other burgeoning mathematicians I was not burdened with unrealistic ambitions. I didn't expect to do great things, I would be happy if I could just get a job and support myself. I was married. I had married very young. At twenty. Far too young.

UP: I always thought that teenage marriages were very romantic.

RH: What a mistake! Anyway we lived in New Jersey in a town near Hackensack called Teaneck. Living in Manhattan was impossible unless you were willing to live in a slum. So I ended up commuting every day across the river.

UP: You were dedicated.

RH: I enjoyed it. And I had the great good luck that Peter Lax offered to be my advisor. He was just one year older than me, he had already been famous for over a decade. He was a Hungarian prodigy at fifteen. Paul Erdos introduced him to Einstein as *a Hungarian math prodigy*. *Why Hungarian?* Einstein asked. Peter showed me his list of problems, and I picked the one that looked the easiest, because it was just algebra. I got nowhere. As a matter of fact, it was a very hard problem that took decades before anyone was able to solve it. In the end I worked on the mixed initial-boundary value problem. At that time a major active field was developing the theory of linear partial differential equations from second to higher order. The classical theory was limited almost entirely to second order. You know about the Laplace, the wave and the heat equations? Of course you do, every mathematician does, even if you don't admit it. Louis Nirenberg did a lot of work generalizing the Laplace equation to higher order elliptic equations. Lax's specialty was hyperbolic equations, generalizing the wave equation, so I got involved in the hyperbolic case. You want to hear all the gory details?

UP: Sure.

RH: Well, my job was to find the most general correct boundary conditions for a general hyperbolic system in a half-space. I was stuck for a long time. Then I ran across a trick in a textbook on applied mathematics. You can combine a Laplace transform in time with Fourier transforms in the unbounded space variables, to reduce the mixed initial-boundary problem in a half-space to an ordinary differential equation in a single spatial variable, the variable orthogonal to the boundary. The resulting ODE problem is a kind of boundary value problem, with one boundary at infinity. This trick, which the textbook used in one particular limited context, could be used in a much more general setting. After that insight, I just had to write it up and show it to Lax. His reaction was *Laplace transform? I haven't used the Laplace transform in years*.

UP: And Laplace transform was Widder's specialty! Quite a coincidence. So it became your thesis?

RH: It was good enough to land me a two-year instructor-ship at Stanford, Way beyond my humble expectations.

UP: So whom did you meet in Stanford?

RH: Polya was there, but I had very little interaction with him at that time. Hörmander was there.

UP: He was visiting Stanford?

RH: He was a regular professor. I had worked on his first book on PDEs and found one silly little misprint, and when I told him so, he looked really disturbed. It took a few seconds for him to see the error – a misspelling of *see* as *se*, which of course is correct in Swedish! Then he looked relieved, and slightly amused. At the time he was teaching a course on several complex variables. That subject wasn't interesting to me, but I was very impressed by the way he handed out typed lecture notes before each lecture. When I expressed my admiration, he said, *There's nothing to it, just go like this*, and pretended typing with his fingers in the air, with a little smile.

UP: It must have come in very handy when he had his book published. I remember it very well. It was in our school-library. I understood nothing. That excited me a lot. I guess he is very efficient.

RH: I ran into him again at Stanford years later, at a week's celebration for Peter Lax turning sixty. He still remembered me. He had put the result of my thesis into his multi-volumed work on PDE. But he made a qualification. He said I had not solved it completely, only in the rough. Reiko Sakamoto had convinced him that she had the first complete proof. That's not true. I solved it, then she did it over again in a much more obscure manner. But who cares? It doesn't matter.

UP: Do you have any other stories about Hörmander?

RH: He gave a talk once while his famous predecessor Arne Beurling was in the audience. Hörmander was doing everything with Fourier transforms, and just to show that he too had a finger in the pie, Beurling asked about doing it with Fourier series. Hörmander quickly and briefly dismissed Beurling's remark as too trivial a case to even mention.

UP: Paul Cohen was there at Stanford too?

RH: Yes, I knew Cohen fairly well. He could be very aggressive. He always wanted to be on top. He would ask you a question and if you weren't prepared to battle with him, you had to either admit defeat or just ignore him. As if he was still a prodigy. He grew up in impoverished circumstances, and was introduced to calculus when he was nine or so. He had been asked by Scientific American magazine for an article about the continuum hypothesis, but they found his contribution impossible to edit for publication. He was looking for help. I did rewrite it successfully, it was published as a joint article. Later on he suggested that we work together on a popular book, but I declined. From a career viewpoint that was probably a mistake, but I just didn't feel comfortable working with him. You know about Courant–Robbins, don't you?

UP: Of course, it's a classic.

RH: Do you know that Robbins wasn't supposed to be listed as a co-author? Courant was the senior author, and he expected sole credit, of course with an acknowledgment to Robbins in the preface. After all, Courant–Hilbert was written by Courant, not Hilbert, with a lot of help from junior authors, and Hilbert was listed as the senior author only as a token of respect. But Robbins forced Courant to list him as a co-author. He got no information about royalties, though. Every once in a while a check would arrive in the mail, with a friendly greeting, but no explanation.

UP: So you ran into Courant when you were at the Courant institute, before it was named Courant?

RH: It was named after him after he retired. I even used a desk where he had once sat. People were really impressed by that. Later on Jerry Berkowitz assigned me as a graduate assistant to work with Courant on the English translation of volume 2 of Courant–Hilbert – Partial Differential Equations. I was just supposed to do copy editing of the galley proofs, but as an experienced editor I couldn't help making occasional suggestions for editorial improvement. This always amused him greatly. Sometimes he accepted my advice. Once I went to a course he was giving. He spoke in such a soft voice that only the people in the front row could hear him. He started out by saying he had sometimes been told that people couldn't always hear him, so would anyone who couldn't hear him please raise their hand, and then he would know he should talk louder. No one raised a hand. Courant was famous for playing up to the rich and powerful. He got a lot of funding that way, and used it very successfully, but some mathematicians turned up their noses at such vulgar behavior.

UP: Anyone else you recall from your Courant days.

RH: There was another Harold Shapiro in addition to the *Swedish* Harold S. Shapiro, I mean Harold N. Shapiro, the number theorist. A loud guy. They used to say, *S is for skinny, N is for noisy.* Do you know what Harold N. did to a promising student of his?

UP: Something unmentionable?

RH: He gave him as a thesis problem the twin prime conjecture. Can you believe it?

UP: I can believe anything.

RH: The poor student! That's the kind of problem you give if you hope to become famous through your student.

UP: The chances would be slim, though.

RH: And the student could be destroyed. Let's go back to Cohen.

UP: The Continuum hypothesis?

RH: You know the story of how it happened?

UP: Cohen had no high regard for logicians, and told them, give me your hardest problem and I will solve it.

RH: And they did! And he went ahead and solved it! Just imagine how those guys must have felt.

UP: End of story.

RH: It hasn't ended yet. Forcing is still keeping the logicians busy. Once he had it solved, Cohen had to go to Princeton to show it to Gödel. He knocked on Gödel's door. Gödel opened the door, peered out, snatched the manuscript and closed the door.

UP: Just like that?

RH: A couple of days later, after Gödel had read the manuscript, Paul was invited inside.

UP: Mathematicians are strange. What do you think of the current fashionable theories of Asperger being so prevalent among mathematicians? Masha Gessen in her recent book on Perelman makes a big deal out of it.

RH: I don't think labeling it as *Asperger's syndrome* helps very much. And in her book, which otherwise I found quite impressive, I thought the section on Asperger's was tasteless and unnecessary. I said so in my review in the Intelligencer.

UP: To me this is just a manifestation of the intolerance for eccentricity. The dictatorship of mediocrity. It is the flip side of genius adulation. The bottom line being that those geniuses may be very clever and all that and do things beyond our conceptions, but they are defective, not to say fatally flawed. It gives some consolation.

RH: Could be.

UP: When I was a beginning graduate student I heard a rumor from a visiting student of logic, that Gödel had already solved the CH. That he in fact knew it much deeper than Cohen, who was humbled.

RH: No, that's not true. After all, there has been a thorough investigation of Gödel's Nachlass.

UP: So Gödel gave Cohen his blessing?

RH: He did. In due time,that is. Cohen became a bit impatient for Gödel's public endorsement. Gödel reassured him and told him to relax.

UP: That sounds very human.

RH: Did you know that Cohen also married a Swedish woman?

UP: Had no idea. Wermer, Shapiro and now Cohen, where will this end?

RH: Cohen met Christina while he was in Sweden to visit the Mittag-Leffler Institute. With that name, there was no way he could pass her off as Jewish. He was very secretive about it. Finally he found a rabbi willing to marry them. Rabbis don't usually consent to marry someone to a non-Jew.

UP: And they lived happily ever after.

RH: Yes, as far as Cohen could be happy. He told me that they were a good couple because they were both childish. But as I said, he wasn't an easy character. At Stanford he usually argued against hiring or promoting anybody. No candidate was ever good enough. He had very few students.

UP: Sarnak was a student of his.

RH: Sarnak is a tough cookie, he could stand up to Cohen. He wrote a eulogy on Cohen in the Notices. Cohen developed some strange disease and died in his early seventies. He had spent decades trying as hard as he could to prove the Riemann conjecture. He actually said to someone I know, *I'll show those bastards I'm not dead yet*. There were four people he considered worth talking to about it. Selberg and Bombieri, I don't remember the other two.

UP: But I do not want to drop this issue of Asperger yet. Regardless or not whether you take the kind of diagnosis seriously, and I believe that it is anyway applied frivolously ignoring some more clinical criteria available; one may perhaps speak of certain character traits of mathematicians.

RH: To tell the truth, most mathematicians are boring. Most of them have no real intellectual interests, they just have a knack for doing mathematics well enough to make a living, teaching the same course over and over, and every now and then coming up with some theorem. It's the same way with artists. You tend to think of them romantically, but most of them are very mundane.

UP: So the vulgar conception of mathematicians as a kind of engineers may not be too far off?

RH: Not too far off. But numerical analysis is looked down on by most mathematicians just because mathematicians want to be above engineers. Peter Lax is an outstanding exception. He combines a great mathematical mind, deeply theoretical and abstract, along with original, effective down-to-earth calculations.

UP: This is supposed to be rather rare.

RH: His interest in computation isn't just to give examples of general principles. He's genuinely interested in it for its own sake, as well as for its practical utility.

UP: What about Polya?

RH: We invited him to speak at New Mexico. People in the department were impressed by my connections at Stanford. Phillips and Cohen came too, but you asked about Polya. He gave two talks. His lecture at the College of Education was called *Let Us Teach Guessing*. He used a problem which I later learned is a special case of *Steiner's problem*. Into how many regions is space divided by five planes chosen at random? You simplify from five planes to four, then three, then two, and from three dimensions to two, and then you guess. He was admirably patient. And pedagogical. For example, he used a ruler to represent a line, and divided it in two with a finger, then divided it once more using a finger on his other hand, then he ran out of hands and used his nose! The audience loved it. Later on I realized how good his books on problem solving are. It was easy to underestimate him. Hermann Weyl once had to comment on him. He said something to the effect that Polya likes to solve nice little problems one after another, but Weyl himself could never work like that. That was unfair. Polya chose important problems. He extracted the essence of some difficulty and presented it very concretely.

UP: I read that Polya considered himself too smart to be a philosopher, but not smart enough to be a physicist, so he chose mathematics.

RH: Replace the word 'smart' with 'good' and you would have the right quotation.

UP: Polya got to be very old, almost a hundred. But not as old as Cartan or Struik. Not to mention Vietoris, who got to be 111.

RH: When did Vietoris stop publishing mathematical papers?

UP: He published some when he was well over a hundred.

RH: Remarkable. Polya spent his last years in misery. He became blind. But he was a wonderful man. Old world civility. We took him to a Mexican restaurant once. He ordered a chile relleno. My wife

cried out to him not to eat the seeds. He smiled and explained that he was Hungarian, he knew very well how to deal with hot stuff. Then his face turned red and his eyes were popping, but he kept smiling. He certainly didn't let on the pain. Wonderful guy.

UP: Going to New Mexico was not the end of your career as a mathematician?

RH: Why should it be? I started to collaborate with a young probabilist, Richard Griego. We got into applications of probability theory to partial differential equations. We wrote a popular article on how to solve the Dirichlet problem using Brownian motion. It was published in Scientific American in the late sixties.

UP: In fact I very much remember that article. It must have been in 1969. I had just finished high-school. I recall my father was intrigued by it.

RH: Happy to hear it. In fact, I have met other people who benefited from it. Griego and I started a new method of probabilistic solution of differential equations. It all started with an obscure paper by Mark Kac, which we realized could be vastly generalized by operator methods. We studied operator differential equations controlled by a stochastic process. Peter Lax gave it the right name, *random evolutions*. Do you want to hear more details? To make sense of what I just said?

UP: Sure.

RH: Well, the main point was to use the central limit theorem from probability as a tool to prove singular perturbation theorems about differential equations. A nice example is to start with a large number of Newtonian particles moving at constant speeds, and suffering collisions which make them switch direction and speed at random. If you put in a couple of properly scaled small parameters, you can make the mean free path between collisions go to zero. Our random evolution model permits us to use the central limit theorem to prove that this system of transport equations, a hyperbolic system, goes to a certain diffusion equation, a parabolic equation, in the limit. In physical language, this is a rigorous proof of the diffusion approximation for a high-density gas.

UP: But then it came to an end. Your career as a research scientist was short but glorious.

RH: You said it, I didn't. It was only about fifteen years. But my work exceeded my expectations. As I already explained. I never expected to become an above average researcher. At the end, there was a paper I was co-authoring. When it came back from being reviewed, I realized that what we were trying to do was essentially routine and uninteresting. Mistakes you can usually correct, often they indicate that you are on to something important and challenging. But not in this case. I apologized to my co-author. I simply couldn't go on with it. It was a tough time. My marriage was falling apart. We had married too young, we had eventually grown apart. I sought and found professional help, trying to sort out my life. And then I fell in love! How wonderful! I felt guilty, of course, but I couldn't help myself. I got divorced, and life started over. As I tell people, and especially you, life begins at sixty. Look at me!

UP: So it was your new love that provided your resurrection?

RH: And also having vibrant intellectual interests that I was passionate about. I had something to think about. And that is the aspect of my resurrection that will interest you.

UP: It was on the philosophy of mathematics and its practice, I understand. What made you into the Reuben Hersh that you are now, and for which people will remember you when all else is gone?

RH: Forget about being remembered, don't expect it. So many people are clamoring to be remembered, and who gets chosen is usually just a matter of chance. But I had an encompassing interest, and I had two talents which are seldom combined, a knack for mathematics and a knack for writing. You seem to have them too.

UP: That is very kind of you to say so. It was 'The Mathematical Experience' which launched you.

RH: I had gotten hooked on philosophy of math when I volunteered to teach a course that was listed in my department's catalogue as *Foundations of Mathematics.* No one had ever offered it, before or since. I expected to just do my usual thing when teaching a subject I know nothing about – pick the best textbook I can find, and stay a chapter ahead of the students. Not this time! All the textbooks I found simply presented three viewpoints – logicist, intuitionist, and formalist, and left it plain that all three were inadequate, unsatisfactory, failures. End of course!! As a teacher I found that situation deeply unacceptable. After all, I ought to at least know what was my own personal philosophy of math. But I found that I simply didn't know. So I had to find out where I stood, what was my understanding of the nature of the subject to which I devoted my life. On my part, *The Mathematical Experience* was a stage in my struggle to figure out my own answers. Then also, my career as a mathematician had given me a special kind of experience, which had not been much exploited in a literary way. I was very lucky to find Phil Davis as a collaborator. We never dreamed that the book would make such a splash. It was far short of our original intentions, but we were desperate and submitted what we had. It seemed only a rag-bag at the time, but nevertheless, it worked, after all!

UP: Yes, I remember very well reading the book in the early 80's. I was impressed by it. I felt that the authors, of whom I had no idea, were really up to something. I also recall Borel at a lunch at the Institute praising the book at the time as a serious work done by people who understood mathematics and what it meant to be a mathematician.

RH: Borel said that? By the way, is Borel related to Emile Borel, the famous French analyst?

UP: Not that I know. I doubt it though. Armand Borel was Swiss for one thing.

RH: The book was reviewed by Martin Gardner. You know him, of course.

UP: Yes. I got a book of his essays translated into Swedish when I was a child. Later I read his columns in the Scientific American. And just a year or so before he died I got into epistolary contact with him. He typed letters the old-fashioned way, even sent me a paper model of a Klein bottle he had made. You cannot do that on e-mail. The reason for getting in touch with him was to refute your anti-Platonist stand in the EMS Newsletter. You may recall the occasion.

RH: I certainly do. You know Gardner believed in God? Literally. He even wrote a chapter advocating the effectiveness of prayer. Gardner was assigned to review our book for the New York Review of Books. Of course that was wonderful. Attention to this kind of book in the NYR, what more can you ask for? You know the NYR, of course?

UP: I have subscribed to it since the mid-seventies.

RH: Good for you. So we have something more in common. Anyway, Gardner liked our book on the whole, but he attacked our anti-Platonist philosophy.

UP: You got a mixed review, in other words.

RH: Gardner was a Platonist. That makes sense for someone who believes in God. If you believe in God, you have an obvious place to put mathematics *out there*. I understand why my anti-Platonism upset or offended him. I never met him personally, but we did have a sort of connection by our common connection to the Scientific American. There were attacks from some other people that I can't so easily tolerate. The worst was from a certain computer list-serve called FOM, meaning Foundations of Mathematics. It belongs to a clique of logicians who not only work on axiomatic set theory, they worship it as The Foundation Of Mathematics. In what sense does mathematics have or need a foundation, let alone what might such a foundation be? I got lured into signing on to this activity. When I refused to convert to their ideology they made me an object of abuse and ridicule. Eventually I escaped by signing off from their computer list. And then, much before all that, there was Professor Hilary Putnam.

UP: A logician at Harvard. Nothing to do with the Putnam exam I take it, although I always made the naive connection when I first encountered his name.

RH: No connection. Jewish mother, WASP father. You know what a WASP is?

UP: I spent several years in the States. In a sense part of my formative ones.

RH: Sorry. You never know. I don't want to take anything for granted. Anyway I had sent a piece on philosophy of mathematics to the Monthly. Putnam was the referee. He referred to my piece as doggerel. I guess he thought of his own stuff as real poetry. As a consequence, The Monthly rejected it, which was a good thing, as Gian-Carlo Rota quickly published it in his journal – the Advances. A much better place for it.

UP: Rota was a dictator.

RH: Sure. And an excellent editor! You should know about that. I've never been an editor. Being one gives you a lot of power, and you need to use it wisely in order to do a good job.

UP: By the way I think of philosophy as the poetry of science. Philosophers do not take kindly to this notion. I mean it as a compliment though. What I mean is that philosophy proceeds by evocation rather than argument, and that it is very important that you present it in an elegant way. Among mathematicians expounding on philosophy I find that Yuri Manin stands out. He is a real pleasure to read.

RH: Manin is great. He has such wide and penetrating interests. Did you ever read his book on logic? It's written from the perspective of a mathematician. And as I understand, written from scratch. He taught himself logic.

UP: Yes. I was very much influenced by it. I came across it when I once tried to teach mathematical logic to undergraduates at Columbia.

RH: The problem with most academic philosophy of mathematics is that it's not about actual mathematics, it's about other philosophers of mathematics, their little clique. They aren't interested in mathematics or mathematicians, they aren't even interested in

regular ordinary philosophers, they are just writing to answer each other and argue with each other. Look at Quine, for God's sake. Very well respected within the logic community. But such an arrogant pedant. He didn't know about the Riemann hypothesis – OK. I can understand that. But what was far worse, he wasn't even interested! The supposed greatest living philosopher of mathematics, and he neither knows nor cares about the most important open problem in mathematics. This man wrote that everything in mathematics can be *got down* to sets. In plain words, to do philosophy of mathematics, it's unnecessary to know anything about mathematics beyond set theory. How ignorant, presumptuous and arrogant! I am too blunt, I know. I have actually met a few philosophers who have a taste for mathematics. and I have finally met one here in New Mexico who is willing to talk and listen to me. Apart from this new acquaintance, there is really no one around here with whom I can discuss those matters, who really wants to listen. Some do it politely for a few minutes. My wife tries to do it, but she can only take so much. It's really exhilarating to have someone who listens as attentively as you. It's wonderful. It makes me blabber, and now I fear I am going beyond all bounds. Are you really going to write all this down?

UP: As much as I will be able to recall. Your powers of recollection are remarkable, once you start unwinding the threads of your memory. So much is retained. Not immediately accessible of course. You have to pull at it. But eventually one thing will lead to another.

RH: Still, it makes me a bit nervous. Where were we?

UP: We were speaking of the ignorance of philosophers when it comes to mathematics. I admit that the more ignorant you are the easier it is to hold firm opinions.

RH: Take Alonzo Church. An important, influential logician, certainly. No question about that. Church wrote down a long formula, involving an X; then he needed another formula identical to the first, except that X was replaced by Y. After mentioning that of course he could simply write something like *let X be replaced by Y*, he decided that the safest thing was to just write the whole thing all over again, but using Y instead of X. Super careful. Incredible. When Gian-Carlo Rota was an undergraduate at Princeton he attended Church's course. Solomon Lefschetz looked into the room, saw Rota sitting there, and shook his head in disapproval. And then, what about Ludwig Wittgenstein? *Mathematics is nothing but calculations. It has nothing to do with concepts or ideas*. How absurd! He is saying such a thing, even while mathematicians are trying hard to explain to him that we are interested in IDEAS ABOUT CALCULATION. With Alan Turing sitting right there in front of him, Wittgenstein is saying mathematics has nothing to do with concepts!

UP: I guess we are in a sense talking about Church's thesis. The point of mathematics is to make sense of calculations and to decide what calculations are to be done.

RH: You can put it that way if you want. Or their idea that mathematics essentially consists of deductive proofs. But in reality, nobody could follow all the way through a completely explicit detailed formal proof of any substantial interesting piece of mathematics. Unless it's a very simple one, like the examples that Hardy pulled out in order to convince people of the beauty and compelling power of mathematics.

UP: It is a commendable ambition.

RH: But misleading.

UP: Very much so. What makes for a convincing argument is not a long deductive chain but the way it fits into the web of mathematics.

RH: Well, in order to include him in my book *What is Mathematics, Really?*, I had to read Wittgenstein.

UP: He seemed very influenced by Russell, thinking of mathematics in a so to speak mechanical way, as a sequence of tautologies. Ultimately this view implies that mathematics contains no new knowledge, everything is in the axioms. It strikes me as somewhat peculiar that the richness of number theory is hidden in the simple axioms of Peano. There seem not to be enough information in them.

RH: That was the early Wittgenstein, the Wittgenstein of the Tractatus. The later Wittgenstein was completely different. He had some good points and some very bad ones. He emphasized that mathematics, like language, is a human activity. Excellent! But he went on to claim that a mathematician is free to do anything he pleases, anything at all. That is not true, it is ridiculous.

UP: So you agree that there are constraints. A mathematician is bound by rules beyond his control.

RH: Exactly. That is the essence of the mathematical experience, as eloquently described by Hardy.

UP: So you do not deny its validity?

RH: Not at all. Why should I?

UP: You have said that mathematics is objective as far as the individual is concerned, and subjective as far as the collective. Would you care to elaborate?

RH: Leslie White was the one who first said that plainly and clearly. Of course mathematics has a very high degree of objectivity. It doesn't matter what is your race, nationality, or religion, root 2 is irrational and pi is transcendental.

UP: So women do not think another mathematics.

RH: No. The Cauchy–Kovalevskaya theorem is neither male nor female.

UP: In what sense is mathematics subjective?

RH: Mathematics is a collective invention, like law or art or language. It's external from the viewpoint of the individual studying it, but it's internal with respect to human culture as a whole. It exists within the shared consciousness of human beings. Of course it's still very different from law, language or art. In particular, mathematics certainly is not just a language, although some people do thoughtlessly say so.

UP: I have a colleague who seriously claims that the difficulties students have with mathematics are linguistic. They have simply not understood 'mathematish' so to speak. They need to have the definitions of mathematics and the formulas translated into plain everyday language. According to this theory some of us instinctively acquire 'mathematish' but the rest need to be explicitly instructed as to its 'grammar' and vocabulary.

RH: That is dumb.

UP: I am glad that you agree. What is worse that this colleague seems to catch the ears of mathematical educators. What about law and art?

RH: Law is about more or less arbitrary regulations and their rational interpretation, and of course that doesn't have the same force as mathematical reasoning. And art, although many mathematicians claim that they are really artists, is likewise softer than down-to-earth mathematics and does not command the same kind of consensus, not even the same kind that law inspires.

UP: When you speak about mathematics are you not really speaking about the practice of mathematics? Mathematics is practiced by human beings, and we do not see it practiced anywhere than by humans, thus the argument that it is a human invention and would not make sense outside humanity is more or less tautological, in the sense of being circular and trivial. And of course what is considered important and beautiful in mathematics is subjective and vulnerable to the forces of fashion. Definitions and concepts are human inventions, but like all inventions, in the mental as well as the physical world, they have unintended consequences.

RH: But you exempt truth?

UP: Yes, I exempt truth. What is true in mathematics is not up to our discretion, certainly not as individuals.

RH: But in practice truth is agreed on by a process of social confirmation. I can give you a specific concrete example. As I told you before, I worked on linear partial differential equations with constant coefficients. My work was later extended by Heinz-Otto Kreiss to the case of variable coefficients. His theorem was quickly accepted as a *known* result that anyone else can freely quote and use. The proof is long and complicated. I could never really understand it all. But in the course of my mathematical education and research there have been many things that I accepted without completely understanding the proof. I would just assume it was my own fault, either I didn't know enough or I wasn't smart enough or I wasn't trying hard enough. Lax decided Kreiss's theorem was true. I don't know for certain how thoroughly he went into it. He knew Kreiss well and had a high opinion of his mathematical work. This particular result fitted well into what one might expect, based on general knowledge of the subject. It used the appropriate tools and methods, it encountered and overcame the expected difficulties. I would expect that he listened to Kreiss explaining it to him in his office until he was convinced. Once Lax decided it was true, no one doubted it. When Kreiss wrote it up for publication in NYU's Communications on Pure and Applied Mathematics, he didn't have to struggle to make every detail clear and explicit. He could publish it in an incomplete, cryptic form, because it had already been accepted by everyone. I suspect that you know of similar examples in your own field.

UP: Sure. One obvious example is Hironaka's resolution of singularities. I doubt that anyone has really gone through all the details. Most people like me, who have appealed to it in their work have not even made the attempt to read the paper, but trust it anyway, because that is socially acceptable. In a way it can be seen as an axiom, something you can rely on without understanding. And an even more generally known example, the proof that there are only 26 sporadic groups. The proof of that, scattered through tens of thousands of journal papers, is too long for any single mind to fathom in all its devilish details. And sure enough, as I understand it, small defects are continually being discovered and fixed, the general idea being that all the mistakes are fixable.

RH: So you agree, even when it comes to truth in mathematics it is a matter of social convention.

UP: But the remarkable thing is that this convention is so consensual. As I have already noted, deductive reasoning is not congenial to humans, when we as referees accept a paper we use other supplementary ways of being convinced. I agree with you that in

practice mathematical truth is based on social consensus. In fact everything you say on the practice of mathematics we agree on. But I think that there is something beyond the practice of mathematics, beyond the human fallible way of doing mathematics. Outside of mathematics, socially accepted truths may be successfully challenged. And even in mathematics, if there is a counter-example to a previously authorized theorem, that will surely trump. Just as in science, our accepted truths are only provisional, although many of them have stood the test of time for a remarkably long time.

RH: Absolutely right. Nevertheless, I hold that the practice of mathematics is all there is to it. I would also emphasize that the most fundamental mathematical practices – counting on your fingers, and spatial intuition – are grounded, like all human activity, in our physical beings, in our bodies and in being in the world. Anything beyond that is mysticism. Wittgenstein's great insight, which was bitterly contested, is that the role of philosophy should be *to show the fly the way out of the fly bottle*. So many philosophical quandaries are illusory and artificial. The fact that language allows a certain question to be asked, by no means implies that a meaningful answer is possible or even conceivable. A famous fascinating question was first asked by Leibniz, and then repeated by Heidegger: 'Why is there something rather than nothing?' It is a useless question. It does not make sense, and no conceivable answer to it could make sense. The only reason that the question is asked is that it is possible to formulate it. Mathematical Platonism is a similar kind of fallacy. It arises from the unfounded idea that there must be something to mathematics beyond the practice of mathematics. You and I can agree on every basic issue of mathematics and disagree on this transcendental issue, which is not even an issue.

UP: What is your position on physical laws? Do they exist or are they just social constructs?

RH: Of course they exist, they are existing social constructs. To talk as if social constructs are things that don't really exist is untenable nonsense. Your electric bill exists, you'd better pay it or you'll be sorry. As to the laws of physics, they are not observed with our eyes or our instruments, they are formulated as part of our effort to make sense of the physical world. That means of course that we can't just make them up any way we please. There is a physical reality out there. The most devout anti-materialist doesn't doubt that his teeth are real, when he is having a really agonizing toothache.

UP: This is of course the standpoint of Karl Popper. Physical theories are just human constructs, but belonging to the objective world of thought – World 3 in fact, to use his somewhat unimaginative terminology, to be distinguished from the World 2 of individual thought and consciousness, all of them distinct from World 1 of the physical world. They are provisional. Theories are only 'true' as long as they are not contradicted. This is inductive reasoning according to Poppers interpretation of induction, which most of his critics do not seem to get. It concords beautifully with R. G. Collingwoods' distinction between deductive and inductive logic, the former is compelling the latter is permitting. Now, Popper failed really to consider mathematics seriously, probably because like most modern philosophers, and here I very much include Wittgenstein, he did not know much about mathematics and had certainly done no work in mathematics, which is a prerequisite for understanding mathematics. Thus he tended to exempt mathematics from science. He did not consider it empirical and thus not liable to the fallacies provided by inductive reasoning. He thought of it as an island of pure and incontestable truth, and hence as somewhat uninteresting. But when it comes to the practice of mathematics we know that it is not really deductive, mathematical truths are also products of social consensus. The difference is that traditional truths of mathematics can be challenged just as traditional beliefs are in science *as I have already mentioned*. And just as in science there are objective ways of coming to a verdict. By objective I mean ways that are agreed on prior to their conclusions. It is not like the case of fashion, when one fashion replaces another, the transformation is incontestable. The new fashion simply takes over as a social force trumping the old one who no longer has any say. This is not the way 'truths' are overthrown in science, although the in my opinion over-rated Thomas Kuhn and his theory of paradigm shifts, seem to imply something like that. Popper is clearer on the issue. The change is through a test. A test is not of universal validity, it is simply designed as to be accepted by two warring parties, by finding so to speak the 'biggest common divisor'. This is democratic. Not in the sense of voting, but always seeking and finding common ground. Popper's vision, and as such it is meta-physical and transcendental, is that there is a 'Truth out there' but we humans will only be able to approximate it. Intrinsic to his vision is that when one approximation replaces another this new approximation will be a 'better one'. Science, as a human enterprise is accumulative and progresses. Unlike the humanities and philosophy changes are not random and frivolous. As Kuhn remarks, and here I agree with him, progress is based on repudiation, by closing off certain lines of thought we are, in my words, able to penetrate deeper into the configuration space of ideas. This is how evolution works.

RH: That was quite a mouthful. I thought I was the one being interviewed, not the one who needs to be lectured to. I have also noticed that Popper seemed to ignore mathematics, putting it on a sort of pedestal. But his student Imre Lakatos applied Popperian thinking to mathematics, and profitably too. His writings on mathematics offended the cliques of academic philosophy of mathematics, and so they didn't get the attention they deserved until long after his death.

UP: Sometimes this is an advantage. Your disciples may propagate your ideas and then you do not have to worry about internal

consistencies, on the contrary the more inconsistently they are presented, the wider the potential audience. Just think of the case of someone like Marx.

RH: Your jokes do enliven the conversation. Science is not the physical world, as I told you, it is our collective attempt to make sense of the physical world. Your notion seems to be that there has to be an actual *Mathematics* playing the role of the physical world, apart from us, residing in some Platonic heaven. And then apart from that transcendental *Mathematics*, there is also the practice of mathematics, which is the human effort to make sense of the inhuman transcendental *Mathematics*. You are the fly that needs to be led out of the bottle. You are seduced by false analogies. Let's make this discussion a bit more concrete, Does infinity exist?

UP: Existence has so many meanings. You can easily get confused.

RH: That's my point. But you know the meaning of the question, even if it's embarrassing to you.

UP: I agree with you that it is a key question, a kind of litmus test when it comes to the Platonic conception of mathematics. Truly it is very hard to manifest infinity in a physical way. Even if the universe would be infinite, which some cosmologists seem to believe, how would we ever verify it? All I can say is that Gauss did not believe in the actual infinity, only the potential.

RH: Long before Gauss, that goes back to Aristotle. It was Georg Cantor who by one sweeping gesture collected all the integers into one set.

UP: This was a very powerful thing to do.

RH: Infinitely powerful, it would seem. But by that very token, clearly illusory. It's one more example of language letting us reify an act which has only verbal meaning. Take the fact that every number can be doubled, so that there are as many even numbers as there are numbers. This was first noticed by Galileo. In the language of set theory, it gives the surprising fact that a subset of a set can be as numerous as the set itself. But all it really says is that every number can be doubled. And this is actually not so easy, if your number consists of a really very, very great many decimal digits. The notion of infinity is really a negative one, not a positive one. It means that we agree to ignore the boundary of the domain we are studying, it's very far away and we can just ignore it. For example, in theory (but not in practice) we can ignore the fact that when numbers get very large they become very difficult to factor. Or in geometry, what is the Euclidean plane but a very, very large sandbox? So big that we never need to draw a circle so big that it hits the boundary. So we can just pretend that there isn't a boundary at all. In fact, the word *infinity* just means *no boundary*. There is no such thing as an infinitude of riches. Imagining that you have collected all the integers, and calling that imagined collection N, does not enable you to take all the numbers under your control.

UP: It is in fact much harder and much more vertiginous to think of very large finite numbers, you know the number of digits of which takes so many digits to write down that it in itself must be expressed by a number with so many digits and so on a number of times the digits of which, you get the idea …

RH: … I get the idea …

UP: … than to think of infinity itself which is trivial.

RH: It is trivial because *infinity* simply says we wish to ignore the boundary. It simplifies, not to say trivializes. We simply ignore technical difficulties. We sweep them under the rug.

UP: The rug which is infinity and which allows everything to be swept under it. Are we not coming full circle?

RH: There is no need to go full circle. Infinity is just a stratagem to simplify our thinking. Mankind will never reach infinity. Why worry about large numbers we will never reach? Surely there is a number M large enough to delimit the ambitions of all humans. If we want to check something, anything, it would be enough to check it up to that number.

UP: Now you are getting carried away. That number M certainly becomes elusive. It is aptly named by the letter M for being meta-physical. It cannot be manipulated like an ordinary number, because it is a meta-physical number. It cannot be specified, at least not by humans, because if specified and pinned down, so would $M + 1$. You remind me of a boy who thinks that numbers are buttons. Through immense diligence and dedication he collects all the buttons in the world and then he says that adding one is impossible, because after all there are no buttons left to add with. What would you say to that boy? That he should start collecting grains of sand instead like Archimedes?

RH: I would have a long, serious conversation with him. Still you must admit that infinity is a pretty slippery concept. And if you don't think so, it's because you're so used to the concept that you no longer find it strange and contradictory as mathematical innocents find it. On the other hand, you think that those incredibly high cardinals, inaccessible, measurable or whatever they are called, that are thought up by logicians, have a transcendent reality? If so, God chooses strange vessels for his insights.

UP: I must admit that I find those things very fishy indeed.

RH: Yes.

UP: So if you deny infinity you deny that there is any meaning to the notion of an infinitude of primes?

RH: Euclid never said *infinity of primes*. He simply showed how, given any collection of primes, you can construct a new one.

UP: Yet even if you believe in the potential infinity as opposed to the actual, you have some faith in an inexhaustible supply. What you are saying is that there are two levels of existence, one potential and one actual. The former somehow weaker than the latter. You are denying the infinitude of the actual but not of the potential.

RH: There is nothing mysterious about that. Accepting infinity is simply agreeing to ignore complications at the horizon by pretending there is none. The same goes for primes. When it comes down to producing an inexhaustible supply of primes, Euclid's method becomes impractical. Humans can make long lists of primes, but I will never be surprised if every such attempt can be superseded.

UP: But that by itself is a testimony to infinity itself, no matter how many occasions, you will never be surprised. It reminds me that a single counter example to a theorem compels you to reject a potentially infinite number of purported proofs sight unseen, admittedly based on the transcendental faith in the consistency of mathematics.

RH: That is interesting.

UP: Now in analysis you are dealing all the time with infinite sets, especially countably infinite. And think nothing about it. Giving an infinite series, any finite sub sum gives no clue as to whether it is convergent or not, in a sense you need to 'see' all the terms to make sense of it. The same with constructions of Cantor sets and other fractal animals. To stop half-way would leave you with something silly, it is only when you go all the way to infinity those creatures become truly interesting. Now the negative result of the uncountability of the reals is the only thing you need to take into account when you are an analyst. Modern measure theory would be impossible without it. Thus in a sense the countable infinities are actualities for the analyst, while the uncountable of reals is merely potential and in a sense metaphysical. To go beyond this in human mathematical practice is simply pointless, no serious mathematics involves anything beyond the continuum. It might be different would we be able to do arguments involving an actual infinite number of steps, then every theorem in number theory could be verified using case by case study. It would be infinitely boring. In a very literal way to boot.

RH: With some care you could easily do away with those countable actual infinities, which are as chimerical as the set of all integers. But I agree that it would be painful. Infinity is just a shorthand designed for convenience. And as to fractals, their applications to nature are suggestive enough. It's really beside the point that on a physical level those structures can't go on indefinitely. The wonders of infinity can be well approximated.

UP: The idea of infinity is very much connected to the desire for immortality. No one wants to live forever, because eternity is an awful long time, yet everyone would like to postpone dying indefinitely.

RH: Speak for yourself.

UP: The idea of your own mortality is a scary concept, especially when you are young. It does not matter whether you live to a hundred, a thousand and even a million, the very idea that you yourself will at some time be at the brink of extinction is what is terrifying. The hidden assumption, which seems so natural when you are young, is the identity of your 'I' over time. This is no trivial assumption, in fact it begs a lot, as you realize when you start to get a more intimate acquaintance with aging. My point is that mathematical concepts such as infinity ties with some very fundamental existential issues.

RH: That only goes to show what I have been trying to say, namely, that mathematical concepts have no transcendental origin, but are perfectly explainable by the human psyche. As to actual infinity, have you ever come across the name of Tipler?

UP: Did he not co-author a book on the Anthropomorphic principle in Cosmos, to the effect that everything in the universe was fine-tuned to prepare the way for the developments of humans, or at least theoretical physicists. I guess this was just within the boundaries of reputable science.

RH: Whether within or not I don't know, certainly he has gone beyond them in later years. I came across a short article of his on the Internet recently. Using some simple physical principles, such as the indestructibility of information and the eventual evaporation of black holes, he predicted with unassailable logic that we humans would all be downloaded into infinite information traveling at the speed of light, all over the place.

UP: This seems like wishful thinking.

RH: Indeed it is. And he becomes really weird when he claims that this final state will be God, and the Christian God to boot.

UP: This shows a certain lack of imagination. It reminds me of an old idea of mine, namely that the past injects into the future, that

no information is lost, that every event no matter how insignificant leaves a tiny trace no matter how elusive and diluted that can be in principle used to reconstruct the event. Otherwise what meaning would there be to say that a thing has occurred in the past, without we having no way of finding out. Psychologically it is easier to imagine that two different causes have the same effect than the same cause having two different effects. It was my way of turning this upside down.

RH: Once again, you are …

UP: … were …

RH: … OK, were the fly in the bottle needing to be led out.

UP: To return to more concrete issues. You recently published a book – Loving and hating mathematics. The very title seems to indicate that your feelings about mathematics are ambivalent.

RH: Aren't yours? Don't you hate it at times?

UP: I guess I have to admit that. I presume that *Loving and Hating Mathematics* is even more focused on the human interaction with mathematics than was *The Mathematical Experience*. Some might say it is gossipy.

RH: I like gossip. Within limits, of course.

UP: This time you co-authored the book with your wife, who is not a mathematician. How did that affect the writing of the book? Was that a major factor in emphasizing the human perspective?

RH: She said, *Let's do something together!* So we had to find a subject that we had in common. In fact, I think it was something that I always wanted to do.

UP: I like to say that you can be very emotional about mathematics, but mathematics offer you no way of expressing your emotions. Maybe this is a clue to the frustration it certainly provokes.

RH: Our book is very much about being emotional about mathematics. What else are loving and hating it? As to not being able to express emotion through mathematics, I am not exactly sure what you really mean by that. I guess to some extent you may be overly influenced by your professed Platonist view of mathematics.

UP: Is it not clear what I mean? Mathematics is completely unconcerned with humans and human emotions.

RH: The standard convention in mathematics is to strictly exclude humans and human emotions from what one writes down. On the other hand, when on occasion someone violates that convention, and their mathematical writing includes something human or even humanly emotional, it often turns out to be very popular and successful!

UP: *But that is exactly my point.* We may leave that topic. Your initial book with Davis was a great success as we have already confirmed. Do you think that this one will be as well?

RH: It's impossible to predict commercial success when it comes to books. If it wasn't, publishing would be so much easier. I could tell you a secret, provided you don't tell anyone, or include it in this interview …

UP: … but if I do not include any names? …

RH: … that might be fine. Anyway, a certain writer published popular columns in a well-known newspaper. When he collected them in a book it was expected to sell very well, but it didn't. As to our latest book, we have participated in a couple of book-signing events here in New Mexico. They were reasonably successful, but we both are known locally. I doubt that we would have such success on a national scale. However, I am trying to enhance the publicity of the book by enlisting U-Tube. If I could get a video on the book propagating on the Web, that would do wonders for its sales.

UP: So you are concerned about the sales of your book?

RH: Don't be so haughty. Just wait until you publish a book. I bet you will find the matter of its sales of utmost importance. Your books are like your children, you wish them every success …

UP: … and your only ticket to immortality?

RH: Speak for yourself. The key is to get a very good video. I had been thinking of using animation, but when it's done by professionals it gets very expensive.[1]

UP: In 'loving and hating' and also in many of your articles you bring up racism in general and anti-Semitism in particular. Is being Jewish very important to you?

RH: Yes and No. I'd like to say No, but there's no getting away from recent history. My memoir on Jews in U.S. mathematics has been chosen for Princeton's next anthology of the best recent articles on math. As I told you, my teen-age ambition was to

[1] In the original version there was a longer digression on this projected video, but as naught come of it, he asked me to delete it as being irrelevant, when I asked for his permission to publish the interview.

fight Hitler. I'm not a Zionist, but my father was. He sent me to Zionist summer camps when I was a child, in order to learn Yiddish, among other things. I resisted his pressure, and learned very little Yiddish. I am very strongly opposed to Israel's policies toward the Palestinians. I have been included on lists of so-called *self-hating Jews*. Concerning religious participation, I feel most at home with Quakers. They try to change the world in a modest and humble way. I very much sympathize with that, even if I often despair. You can't always despair. Sometimes you have to force yourself to be optimistic, to feel that you can make a difference in the world.

UP: As to the notion of races in general and Jewishness in particular, is that not mere social constructs. When it comes to races one can at least try and base it on some objective criteria such as DNA. But that does not work for deciding who is a Jew or not.

RH: Social constructs can have nasty consequences.

UP: I also think that concomitant with the kind of xenophobia we associate with racism there is also a sentimental fascination for exotic elements among your ancestors. Both tendencies can probably be found in many individuals, testifying to the intrinsic inconsistencies in our desires. I myself harbor some hopes that I may have Lappish blood (or more precisely mitochondria). Likewise to follow the historical flow of ancient populations, as reflected in present day DNA or cultural traits is a fascinating exercise, although it has by some been attacked as racism. Then humans are as a species remarkably genetically uniform, supposedly as a consequence of a fairly recent bottle-neck which almost wiped us out. It is noteworthy that any child can learn to speak the prevalent tongue without accent regardless of race …

RH: That was another mouthful. What are you really up to? I thought you were trying to bring up anti-Semitism.

UP: By all means. I am thinking of George Birkhoff. He was notorious was he not?

RH: Not to be unfair, but he was a real bastard.

UP: But he was not alone in the States at the time. Anti-Semitism, if in a relatively milder form was rampant, just as assumptions about the inferiority of the blacks.

RH: That's right, he wasn't alone. Some people were anti-Semitic, some were not. James Alexander, the topologist at Princeton, used his upper-class connections to force the Princeton administration to hire Solomon Lefschetz as a professor of mathematics. Unique in 1923, a Jewish professor in such an Ivy League college. At Columbia, Cassius Jackson Keyser was instrumental in hiring their first Jewish math professor, Edward Kasner. Their example shows that there was a choice whether to be anti-Semitic or not. Of course, Birkhoff had a theory. He explained that Jews mature earlier, and hence gentiles should be protected against them.

UP: It was a kind of affirmative action.

RH: Thank you for another amusing comment. James reports a conversation between Birkhoff and an officer of the Rockefeller Foundation, who noted afterwards, '*B. speaks long and earnestly concerning the Jewish question and the importation of Jewish scholars. He has no theoretical prejudice against the race and on the contrary every wish to be absolutely fair and sympathetic. He does however think that we must be more realistic than we are at present concerning the dangers in the situation and he is privately and entirely confidentially more or less sympathetic with the difficulties of Germany. He does not approve of their methods, but he is inclined to agree that the results were necessary.* No doubt he didn't know that within a few years the results would be the murder of millions of men, women and children, including nearly three dozen of my own cousins. Here's a funny story about Birkhoff that is certified by someone I know who knows someone who was there when it happened. Birkhoff actually was trying to get Rochester to hire a Jewish refugee mathematician. They refused. He replied in anger, *Who do you think you are, Harvard?* You get the joke?

UP: Not really.

RH: For a second-rate university like Rochester, it was pretentious to be anti-Semitic. For an elite institution like Harvard, it was only natural.

UP: It would have been different if he had been Jewish. Then he would have been classified as self-hating and been forgiven as an eccentric.

RH: What on earth are you talking about? Forgiven by who? Are you serious, or just baiting me? But I gladly admit that Birkhoff's anti-Semitism was nowhere near as bad as Hitler's.

UP: If he had been exposed to the Nazi variant he may have changed his views, as many moderate anti-Semitics did after the war. No one has done as much as Hitler as to discredit anti-Semitism. But at what a price!

RH: Yes, we must thank Dear Adolf for that. Birkhoff lived until 1944. So, to be fair, by then he may no longer have been *inclined to agree that the results were necessary*. For all we know, he might have voted against the Holocaust. When Ralph Phillips wrote about Birkhoff's active malignant influence, Saunders Mac Lane, who collaborated with Birkhoff's son Garrett, was sufficiently irritated

to write an article in defense of Birkhoff. No surprise – his defense was, *It's not fair to single him out, everybody was like that in those days*. But then, to be fair, maybe MacLane never heard of Keyser or Alexander.

MacLane studied in Göttingen during the 30's. He later reported that he had experienced nothing untoward. It would be honest to write and report that at the time you didn't notice anything wrong, now you know that you were badly mistaken, blind to what was going on. I could respect that. But to still pretend after all those years that nothing was really bad, because you didn't notice it, that's bizarre, to put it politely.

UP: In retrospect, for obvious reason, we tend to emphasize the anti-Semitic elements in early Nazi propaganda. I do not believe, pace Goldhagen, that this was what attracted people to Nazism at the time. Anti-Communism I think was a far more serious factor. I guess that the anti-Semitic rhetoric was more of an embarrassment.

RH: So let's not be angry at those early Nazi-supporting voters, they may really just have been premature anti-communists. Well, to be fair, they got what they wanted, and a little bit more. War against Russia, yes! And the battle of Stalingrad! Destruction of the Reichswehr! Suicide of Adolf Hitler! A communist dictatorship over half of die Heimat! And the murder of my grandparents, whom I never met. Murdered, wantonly and openly, in Vinnitsa, Ukraine, in 1945. If it's not unfair or off-subject to say so.

UP: People cast their votes for all kinds of silly reasons. I would not be surprised that Hitler got votes because he was a vegetarian. Yes supporters, whatever their motivations, obviously have a moral responsibility and there is all the reason to be angry at them. (Angry by the way is a mild word, it holds out the possibility of forgiveness, you may want a stronger.) Yet if you are searching for psychological explanations, it is fully legitimate to look beyond the obvious ones such as anti-Semitism.

RH: Did you know that Nevanlinna was a Nazi?

UP: Osmo Pekonen at Math Intelligencer told me.

RH: Yes, Pekonen wrote about it. That was instructive, and somewhat courageous of him. Nevanlinna was not only a Nazi, he was a Nazi who made up a story claiming he had saved a Jew! You know the story about André Weil visiting Finland, being accused of being a spy, about to be executed, when Nevanlinna saved him?

UP: Yes, I do. I recall being told about it by Ahlfors wife, long before it appeared in print. What about it?

RH: Not to be unfair, it was a lie. Nevanlinna just made it up after the war, to make himself look a little better.

UP: He fooled Weil!

RH: He fooled everybody.

UP: Not Pekonen.

RH: He wasn't even born then.

UP: What is your next project about?

RH: I'm starting to write a biography of my old advisor Peter Lax. What I really want to do is to write his autobiography. To make his life and work really come alive. I don't know whether I'm up to the challenge. I've never done anything like this before.

UP: But you have dreamt of doing it. Come on, you were a budding writer once. Now rise to the occasion. It must be very exciting. You can do much more than you think.

RH: Thank you.

UP: And I would advise you to title the book 'The autobiography of Peter Lax' and have you as the sole author.

RH: That was already done by Gertrude Stein. She wrote *The Autobiography of Alice B. Toklas*.

UP: Maybe we should stop now. You must be exhausted.

RH: I am not. I could keep on talking for ever.

UP: Potentially or actually?

RH: Actually of course.