## Research-data management planning in the German mathematical community

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*In this paper we discuss the notion of research data for the field of mathematics and report on the status quo of research-data management and planning. A number of decentralized approaches are presented and compared to needs and challenges faced in three use cases from different mathematical subdisciplines. We highlight the importance of tailoring research-data management plans to mathematicians' research processes and discuss their usage all along the data life cycle.*

## 1 Introduction

Scientific progress heavily relies on the reusability of previous results. This in turn is closely linked to reliability and reproducibility of research, and to the question whether another researcher would arrive at the same result with the same material. In mathematics proofs, together with references to definitions of mathematical objects and already verified theorems, traditionally contained all the information needed in order to verify results. However, the advent of computers has opened up new resources previously deemed impossible, while increasing the need for well-adapted research-data management (RDM). For example, algorithms are now implemented to arrive at new conclusions. The size of examples has exploded several orders in magnitude. And some proofs have become too complicated for even the brightest minds, such that software is consulted for thorough understanding and ver-ification.<sup>1</sup> Studies [\[12,](#page-5-0) [13,](#page-5-1) [18,](#page-5-2) [19\]](#page-5-3) from various fields of applied mathematics show that nowadays many results cannot be easily reproduced and hence verified.

As we outline in Section [2,](#page-0-1) there are research data in all subdisciplines of mathematics that need responsible organization and documentation in order to ensure they are handled according to the FAIR principles [\[25\]](#page-6-0) for sustainable, reproducible, and reusable research. One way to achieve this is via a tailored research-data management plan (RDMP), describing the data life cycle over the

course of a project [\[17\]](#page-5-4) and providing guidance to fulfill funding requirements.<sup>2</sup> In mathematics, it is particularly important to treat the RDMP as a living document [\[4\]](#page-5-5) because the mathematical research process is hardly projectable and does usually not follow a standardized collection–analysis–report procedure. In subfields with experience in using such documentation, three-fold reports – at the grant-application stage, as a working document, and as a final report – have proven useful. We discuss this in Sections [3](#page-1-0) and [4,](#page-2-0) spotlighting examples from different subfields, and conclude this article by listing central topics for RDMPs in all areas of mathematics.

## <span id="page-0-1"></span>2 Mathematical research data

Following [\[9,](#page-5-6) p. 130], we define *research data* as all digital and analog objects that are generated or handled in the process of doing research.<sup>3</sup> In mathematics, research data thus include paper publications and proofs therein as well as computational results, code, software, and libraries of classifications of mathematical objects. A non-exhaustive list of possible formats and examples is presented in Table [1](#page-1-1) and Section [4.](#page-2-0) The apparent diversity of mathematical research-data formats is also reflected in other characteristics, such as their storage size, longevity, and state of standardization [\[3,](#page-5-7) [8,](#page-5-8) [10,](#page-5-9) [22,](#page-6-1) [24\]](#page-6-2), leading to RDM needs and challenges that are very specific to the discipline of mathematics.

One of the most apparent challenges is the question what metadata are sufficient for reusability. We will answer this question partially for the mathematical subfields presented in Section [4.](#page-2-0)

<span id="page-0-0"></span><sup>1</sup> See, e.g., the story outlined in [https://xenaproject.wordpress.com/](https://xenaproject.wordpress.com/2020/12/05/liquid-tensor-experiment) [2020/12/05/liquid-tensor-experiment,](https://xenaproject.wordpress.com/2020/12/05/liquid-tensor-experiment) solved with the lean project [https://github.com/leanprover-community/lean-liquid.](https://github.com/leanprover-community/lean-liquid)

<span id="page-0-2"></span><sup>&</sup>lt;sup>2</sup> E.g., at the European level [https://ec.europa.eu/info/funding-tenders/](https://ec.europa.eu/info/funding-tenders/opportunities/docs/2021-2027/horizon/guidance/programme-guide_horizon_en.pdf) [opportunities/docs/2021-2027/horizon/guidance/programme-guide\\_](https://ec.europa.eu/info/funding-tenders/opportunities/docs/2021-2027/horizon/guidance/programme-guide_horizon_en.pdf) [horizon\\_en.pdf,](https://ec.europa.eu/info/funding-tenders/opportunities/docs/2021-2027/horizon/guidance/programme-guide_horizon_en.pdf) and at the German national level [https://www.dfg.de/](https://www.dfg.de/download/pdf/foerderung/grundlagen_dfg_foerderung/forschungsdaten/forschungsdaten_checkliste_de.pdf) [download/pdf/foerderung/grundlagen\\_dfg\\_foerderung/forschungsdaten/](https://www.dfg.de/download/pdf/foerderung/grundlagen_dfg_foerderung/forschungsdaten/forschungsdaten_checkliste_de.pdf) [forschungsdaten\\_checkliste\\_de.pdf](https://www.dfg.de/download/pdf/foerderung/grundlagen_dfg_foerderung/forschungsdaten/forschungsdaten_checkliste_de.pdf).

<span id="page-0-3"></span><sup>&</sup>lt;sup>3</sup> This is in line with the notions employed by the DFG [https://www.dfg.](https://www.dfg.de/foerderung/grundlagen_rahmenbedingungen/forschungsdaten/index.html) [de/foerderung/grundlagen\\_rahmenbedingungen/forschungsdaten/](https://www.dfg.de/foerderung/grundlagen_rahmenbedingungen/forschungsdaten/index.html) [index.html](https://www.dfg.de/foerderung/grundlagen_rahmenbedingungen/forschungsdaten/index.html), forschungsdaten.info [https://www.forschungsdaten.info/](https://www.forschungsdaten.info/themen/informieren-und-planen/was-sind-forschungsdaten) [themen/informieren-und-planen/was-sind-forschungsdaten,](https://www.forschungsdaten.info/themen/informieren-und-planen/was-sind-forschungsdaten) and the MPG [https://rdm.mpdl.mpg.de/introduction/research-data-management,](https://rdm.mpdl.mpg.de/introduction/research-data-management) e.g.

<span id="page-1-1"></span>

*Table 1.* Mathematical research data come in a variety of data formats. Updated table based on [\[24,](#page-6-2) pp. 26–27].

However, as the authors of [\[3\]](#page-5-7) note, 'the meaning and provenance of [mathematical research] data must usually be given in the form of complex mathematical data themselves.' It is thus not surprising that there is no common, standardized metadata format yet. A search in the RDA Metadata Standards Catalog<sup>4</sup> at the time of writing reveals five hits, four from a subfield of statistics and one from economics, none of which could encode information about, say, a computer-algebra experiment. This lack of standardization is in contrast to other disciplines such as the life sciences, where the OBO Foundry<sup>5</sup> hosts more than one hundred interoperable ontologies to describe and link research results, including common naming conventions [\[1,](#page-5-10)[20\]](#page-5-11).

Another important aspect of mathematical research data is its particular data life cycle. Again in contrast, for instance, to the life sciences, where older results can be overruled by new evidence, mathematical results that have been proven true remain true indefinitely. Since they cater for other disciplines such as the physical, social, health or life sciences [\[24,](#page-6-2) Fig. 1 and discussion], mathematics has a particular responsibility to science to preserve their results in a sustainable manner. We discuss this aspect and how mathematics can be embedded in interdisciplinary research pipelines in more detail in Sections [4.1](#page-2-1) and [4.2.](#page-3-0) Section [4.3](#page-3-1) stresses the role thorough documentation plays in this context, using classifications as an example.

## <span id="page-1-0"></span>3 Status quo of RDMPs in mathematics

In the narrower sense of data (rather than research data), it is a common claim in the community at the time of writing that mathematics rarely produces data $<sup>6</sup>$  and that the few data available</sup> need no particular management.<sup>7</sup> This is often based on an interpretation of data being something computational, and mathematics

<span id="page-1-2"></span><sup>4</sup> <https://rdamsc.bath.ac.uk/subject/Mathematics%20and%20statistics>

<span id="page-1-3"></span><sup>5</sup> <https://obofoundry.org/principles/fp-000-summary.html>

<span id="page-1-5"></span><span id="page-1-4"></span><sup>6</sup> Usually, only statistics is mentioned as a data-producing subdiscipline, see, e.g., [https://wissenschaftliche-integritaet.de/kommentare/software](https://wissenschaftliche-integritaet.de/kommentare/software-entwicklung-und-umgang-mit-forschungsdaten-in-der-mathematik)[entwicklung-und-umgang-mit-forschungsdaten-in-der-mathematik.](https://wissenschaftliche-integritaet.de/kommentare/software-entwicklung-und-umgang-mit-forschungsdaten-in-der-mathematik) 7 See, e.g., the unofficial document [https://www.math.harvard.edu/media/](https://www.math.harvard.edu/media/DataManagement.pdf) [DataManagement.pdf](https://www.math.harvard.edu/media/DataManagement.pdf).

being a discipline which is very much paper rather than computer based. Anecdotal evidence suggests that this view is widely established, that there is little knowledge about general RDM, that existing local facilities are hardly used, and that RDMPs are not a standard tool at any stage of the research process.<sup>8</sup> The proposal [\[24\]](#page-6-2) has identified the need to build common infrastructures for all subdisciplines of mathematics, and mathematics-specific DFG guidelines for FAIR research data will be developed in the foreseeable future.<sup>9</sup>

Now, the question of what these guidelines should be is not trivial. A large number of questions from a general RDMP catalogue,<sup>10</sup> are irrelevant for a community which produces foremostly theoretical results. For instance, for mathematicians the cost of producing data is rarely relevant – unlike, e.g., in the life sciences where data might have to be collected in the field. In the same vein, ethical or data-protection questions most often do not play a role, save for, for instance, industry collaborations or studies conducted in didactics. Large parts of the community have little training in legal aspects as, for example, formulae cannot be assigned proprietary rights. In order to avoid the impression that thus all general RDMP questions apply only to sciences different than mathematics, it is imperative to design bespoke catalogues of questions. These should (a) use unambiguous language, for instance, using the term 'research data' rather than the more specific 'data' which many mathematicians do not handle in their research, and (b) avoid superfluous topics while at the same time including sufficient detail, for instance, for mathematics' metadata and preservation needs identified in the previous section. Now, rather than endeavoring to find a one-size-fits-all solution, in the subsequent section we identify important RDM questions for a number of use cases which are known to the authors – focusing on metadata, software, data formats and size, versioning, and storage – and provide those with what we consider to be sensible answers. We use the remainder of this section to report on two RDM solutions implemented in DFG-funded Collaborative Research Centers (CRC).

The CRC 1456 'Mathematics of Experiment' includes 17 scientific projects in applied mathematics, computer science, and natural sciences such as biophysics and astronomy, aiming to improve the analysis of experimental data. The research data here are extremely diverse (e.g., mathematical documents, notebooks, programs, simulation data or experimental measurements) and their handling is

supported by the CRC's dedicated infrastructure project. In regular RDM meetings, four themes are recurrent. First, reusage scenarios: especially in interdisciplinary research the same datasets may be processed or used by different groups; documentation, curation, and publication should be tailored to those groups' needs. Second, reproducibility, both computationally and practically in data recreation. Third, metadata: finding accurate descriptors to help the user understand cross-scientific research data. And fourth, visibility: receiving recognition for stand-alone research data beyond a journal publication is hard. This last topic is usually not part of a standard set of RDMP questions but aims to provide an incentive to increase the effort in research-data creation, publication, and curation.

The CRC 1294 'Data Assimilation' includes 15 interdisciplinary research projects focusing on the development and integration of algorithms, e.g., in earthquake prediction, medication dosing, or cell-shape dynamics. Researchers are thus confronted both with diverse research data and varying cultural data-handling habits. A central project supports their RDM, and IT infrastructure to facilitate collaborative work and knowledge perpetuation to advance good scientific practice is provided. In particular, the CRC designed an RDMP template in collaboration with the University of Potsdam's research-data group. This covers policies and guidelines, legal and ethical considerations, documentation, and dataset-specific aspects. A vital component of the training is then the classification of the digital objects that are reused and created by the individual researchers. This helps them to develop tailored strategies to improve the quality and reproducibility of published results and to sensitize their research-data handling throughout the data life cycle.

## <span id="page-2-0"></span>4 Use cases

We now consider four very different mathematical use cases and discuss their particular research-data needs. These use cases have been identified by the different mathematical subfields in [\[24\]](#page-6-2) as particularly representative for the research community. Central in these expositions for us is to find out how, using RDMPs, we can provide the best, case-specific guidance to make a project reusable.

## <span id="page-2-1"></span>4.1 Applied and interdisciplinary mathematics

In numerous scientific fields real-world problems are simplified, e.g., to experiments, and subsequently described in abstract ways using mathematical models. If a model is combined with input data, it forms a concrete instance of such a problem. With the help of algorithms, the input data are then transformed into output data. Following validations, the interpretation of outputs provides the solution of the initial problem in a so-called Modeling– Simulation–Optimization workflow [\[24,](#page-6-2) p. 77]. For complete RDM,

<span id="page-2-2"></span><sup>&</sup>lt;sup>8</sup> In fact, RDMPs became compulsory in DFG-funding applications only in March 2022, and there are no statistics available on how many mathematics proposals included such a document. See also [https://www.](https://www.dfg.de/foerderung/info_wissenschaft/2022/info_wissenschaft_22_25/index.html) [dfg.de/foerderung/info\\_wissenschaft/2022/info\\_wissenschaft\\_22\\_25/](https://www.dfg.de/foerderung/info_wissenschaft/2022/info_wissenschaft_22_25/index.html) [index.html.](https://www.dfg.de/foerderung/info_wissenschaft/2022/info_wissenschaft_22_25/index.html)

<span id="page-2-3"></span><sup>9</sup> [https://www.dfg.de/foerderung/info\\_wissenschaft/2022/info\\_](https://www.dfg.de/foerderung/info_wissenschaft/2022/info_wissenschaft_22_25/index.html) [wissenschaft\\_22\\_25/index.html](https://www.dfg.de/foerderung/info_wissenschaft/2022/info_wissenschaft_22_25/index.html)

<span id="page-2-4"></span><sup>&</sup>lt;sup>10</sup> For instance, the current questionnaire supplied by the DFG-funded research-data management organiser RDMO [https://github.com/](https://github.com/rdmorganiser/rdmo-catalog/releases/tag/1.1.0-rdmo-1.6.0) [rdmorganiser/rdmo-catalog/releases/tag/1.1.0-rdmo-1.6.0.](https://github.com/rdmorganiser/rdmo-catalog/releases/tag/1.1.0-rdmo-1.6.0)

such workflows should be documented in detail as part of an RDMP.

A standard RDMP questionnaire includes some guidance for the documentation of workflows, such as the main research question, involved disciplines, tools, software, technologies, processes, research-data aspects, and reproducibility. Using this as a template, a tailored questionnaire is currently being developed within the framework of MaRD $11$ <sup>1</sup> to document workflows in detail. This is divided into four sections dealing with the problem statement (object of research, data streams), the model (discretization, variables), the process information (process steps, applied methods), and reproducibility. It is aimed at all disciplines and differs only slightly in whether a theoretical or experimental workflow is documented. The central element of the questionnaire is to establish connections between different steps of the research process in order to improve interoperability of research data. The description of an individual process step, for example, requires the assignment of the relevant input and output data, the method and the (software) environment. At the same time, the documentation of the methods, software, input and output data requires persistent identifiers (e.g., Wikidata, swMATH, DOI) in addition to topic-dependent information.

We consider the documentation of a concrete workflow combining archaeology and mathematics as an example. This is based on [\[11\]](#page-5-12), was created by Margarita Kostre independently afterwards, and described in personal communication as 'very helpful for the reflection of the own work.' The author commented that she will use workflow documentation in the future again, as she believes it facilitates interdisciplinary communication, e.g., about the status of a project, its goal, and data transfer, it provides better clarity in larger collaborations and allows colleagues to enter a project more easily. The aim of this work is to understand the Romanization of Northern Africa using a susceptible infectious epidemic model. On the process level, the workflow starts with data preparation, e.g., collecting, discretizing, and reducing archaeological data. Once a suitable epidemics model is found, the inverse problem is solved to determine contact networks and spreading-rate functions. Subsequent analysis allows the identification of three different possibilities of the Romanization of Northern Africa. The detailed documentation can be found on the MaRDI Portal[.12](#page-3-3)

## <span id="page-3-0"></span>4.2 Scientific computing

While research in pure mathematics strives to determine an ultimate truth, applied or computational mathematics in majority need to deal with approximations to reality: models are usually expressed in terms of real or complex numbers and only finite subsets of these can actually be implemented on computer hardware. Consequently, the result of a computation depends on the format of the finiteprecision numbers used and on the specific hardware executing the computations, making a detailed documentation of the computerbased experiment crucial and reusability of code a must-have [\[5\]](#page-5-13). Thus, the input data and results of a computer experiment and also the precise implementation (code, software, and hardware) of the algorithms used constitute important research data.

Absence of such details in documentation makes applied mathematics face the same reproducibility issues (e.g., [\[2\]](#page-5-14)) as other scientific fields. Still mathematical algorithms make up the foundation of many computational experiments, for instance as solvers for linear systems of equations, eigenvalue problems, or optimization problems, and are thus at the heart of science today. This responsibility calls for rigorous RDM and documentation in RDMPs.

The main difficulty in establishing RDMPs in scientific computing seems to be in creating incentives to adhere to common standards. In case of a single multi-author paper within a larger project cluster, there are two levels to this question: the funding context and local RDM. Regarding the first, incentives should clearly address reporting requirements and incorporate rewards for sustainable RDM, rather than merely counting publications and citations, to ensure the cluster can *stand on the shoulders of giants* instead of *building on quicksand*. The beneficiaries here are other researchers in the project and world-wide. Consequently, global RDM needs to answer what is reported where and why. Regarding the local context, for the collaborative work of the authors, incentives are far more evident. Thorough RDM, documented in a living RDMP, not only accelerates the paper writing, it also improves the reusability of information for future endeavors of the individual authors. Questions center around 'When is the code/data provided? Where in the (local) infrastructure is it stored? By whom? Who is processing it next?'

Consequently, RDMPs should be modularized to enable the single modules to change at their appropriate pace. While the global management rules of a project cluster may not change at all, or at best very slowly, the findings in a single work package may alter the RDMP and thus RDM needs high agility to react to changes. For the software pipeline of an example paper that means: a task-based RDMP, updated as the pipeline evolves, needs to fulfill the requirements of [\[5\]](#page-5-13), while for the project cluster sustainable handover, following (e.g., [\[6\]](#page-5-15)), needs to be addressed in the overarching RDMP.

## <span id="page-3-1"></span>4.3 Computer algebra and theoretical statistics

Large parts of the German mathematical community consider themselves as not doing applied work. This includes fields such as geometry, topology, algebra, analysis or number theory, and also mathematical statistics, for instance. However, these researchers increasingly use computers, too, to explore the viability of proof strategies, test their own conjectures or refute established ones.

<span id="page-3-2"></span><sup>11</sup> <https://www.mardi4nfdi.de>

<span id="page-3-3"></span><sup>12</sup> [https://portal.mardi4nfdi.de/wiki/Romanization\\_spreading\\_on\\_historical\\_](https://portal.mardi4nfdi.de/wiki/Romanization_spreading_on_historical_interregional_networks_in_Northern_Tunisia) [interregional\\_networks\\_in\\_Northern\\_Tunisia](https://portal.mardi4nfdi.de/wiki/Romanization_spreading_on_historical_interregional_networks_in_Northern_Tunisia)

As a consequence, *classifications*, the systematic and complete tabulation of all objects with a given property, grow wildly in size and complexity. They give a complete picture for some aspect of a theory and may be used in many ways, from the search of (counter)examples over building blocks for constructive proofs to benchmark problems.

For instance, the L-Functions and Modular Forms Database (LMFDB) [\[23\]](#page-6-3) contains over 4.8 TB of data relating objects conjectured to have strong connections by the Langlands program: number fields, elliptic curves, modular forms, L-functions, Galois representations. It includes tens of millions of individual objects and stores the relations between these. Entries contain detailed information on reliability, completeness, and several versions of the code needed to compute them. The database has a public reporting system which allows all users to have visibility of any issues or errors.<sup>13</sup>

Computing mathematical objects for classification can often be algorithmically hard and time-consuming. But once computed, results are final and independent of the software used. With larger computer clusters and better algorithms, it is unreasonable and unsustainable to expect researchers who want to build on existing research to repeat individual computations. This expected reuse increases the need for responsible RDM and triggers challenges which need to be addressed in an RDMP. In particular, four themes are central in this regard. First, how can researchers ensure that their research data are correct and complete? Is the connection of mathematical theory and code sound? Second, how can other researchers access, understand, and reuse the research data? Third, how can one ensure longevity of their research data? And fourth, how can researchers report errors/corrections and upload new versions of research data if necessary?

These questions are neatly addressed in the LMFDB mentioned above. To show how things can go wrong without proper RDM, we discuss a classification of all conditional independence structures on up to four discrete random variables, originally published in a series of papers [\[14](#page-5-16)[–16\]](#page-5-17). Of the 2 <sup>24</sup> = 16 777 216 *a priori* possible patterns of how four random variables can influence each other, only 18 478 ( $\approx$  0.11%) are realizable with a probability distribution. Šimeček, the author of [\[21\]](#page-6-4), digitized this result and then left the field after his PhD in 2007. His research data was deleted in 2021 from his former institute's website – the only public place which ever held the database.<sup>14</sup> It was encoded in a packed binary format which is hard to read, search, and reuse. Some files supporting the correctness of the classification for binary distributions use an unspecified, compiler-specific binary serialization format for

<span id="page-4-0"></span><sup>13</sup> Other classification databases targeted at specific audiences are listed at [https://mathdb.mathhub.info.](https://mathdb.mathhub.info)

<span id="page-4-1"></span>14 A backup is still available on the Internet Archive at [http://web.archive.](http://web.archive.org/web/20190516145904/http://atrey.karlin.mff.cuni.cz/~simecek/skola/models/) [org/web/20190516145904/http://atrey.karlin.mff.cuni.cz/~simecek/skola/](http://web.archive.org/web/20190516145904/http://atrey.karlin.mff.cuni.cz/~simecek/skola/models/) [models/.](http://web.archive.org/web/20190516145904/http://atrey.karlin.mff.cuni.cz/~simecek/skola/models/)

floating-point data.<sup>15</sup> The programs used for the creation and inspection of the database were written in a dialect of the Pascal programming language, which has not been maintained since 2006. The sparse documentation is in Czech.

This situation can only be fixed by recreating the database from scratch, including proofs. An RDMP for this project should emphasize the need to list and document each step of redoing the computations, the use of standard data formats with rich metadata for interoperability and searchability of the database, and ensure future reusability of Šimeček's results.

## 5 Discussion and outlook

The problem of reusability strongly relates to a phenomenon called *dark data*, which 'exists only in the bottom left-hand desk drawer of scientists on some media that are quickly aging' [\[7\]](#page-5-18). If research data are not available, they are of course neither traceable nor reusable or FAIR. This phenomenon extends from lost USB sticks and conflicting cloud-based collaboration tools like Dropbox<sup>16</sup>nd Overleaf<sup>17</sup>ithout local backup to papers containing very condensed complicated proofs that can only be taken up in future work if access to handwritten notes of the authors is also possible. A prime example of this is presented in Section [4.3](#page-3-1) where unavailable research data is in stark contrast to the everlasting truth of mathematical results. RDMPs are a tool of choice against such issues, serving as a basic measure to organize the full data life cycle.

From the three case studies considered in Section [4,](#page-2-0) we derive that RDMPs in mathematics in particular (a) stimulate reflection, clarity, and interdisciplinary communication, (b) require flexibility and modularization as living RDMPs, and (c) facilitate the documentation of iterative computational processes by fostering researchdata interoperability and reusability.

We further conclude that archiving and preservation is key in any mathematical subdiscipline. As a very first step to improve the status quo, all research results necessary for reusability (data, code, notes, …) should be stored in a sustainable and findable manner, using resources already documented in an RDMP before a project starts. Ideally, in a second step, citable repositories with persistent identifiers for these research data can be chosen, and, in a third step, these can be annotated with interlinked metadata, implemented via knowledge graphs. Because of the diversity of mathematical research data, the choice of metadata should be made carefully with possible reusage scenarios and interest groups in mind, also documented in an RDMP. If code is part of a publication, thoughts should be given to the detail of documentation

<span id="page-4-2"></span><sup>&</sup>lt;sup>15</sup> A set of scripts for reading these files is available at [https://github.com/](https://github.com/taboege/simecek-tools) [taboege/simecek-tools.](https://github.com/taboege/simecek-tools)

<span id="page-4-3"></span><sup>16</sup> <https://www.dropbox.com>

<span id="page-4-4"></span><sup>17</sup> <https://www.overleaf.com>

and again appropriate citable long-term repositories. In addition, an RDMP should be used as a tool to identify legal constraints, like the compatibility of software licenses, before any actual work is conducted.

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## *References*

- <span id="page-5-10"></span>[1] R. Arp, B. Smith and A. D. Spear, *[Building Ontologies with Basic](https://doi.org/10.7551/mitpress/9780262527811.001.0001) [Formal Ontology](https://doi.org/10.7551/mitpress/9780262527811.001.0001)*. MIT Press, Cambridge, MA (2015)
- <span id="page-5-14"></span>[2] W. Bangerth and T. Heister, Quo vadis, scientific software? *SIAM News* 47, 8–7 (2014)
- <span id="page-5-7"></span>[3] K. Berčič, M. Kohlhase and F. Rabe, [\(Deep\) FAIR mathematics.](https://doi.org/10.1515/itit-2019-0028) *it – Information Technology* 62, 7–17 (2020)
- <span id="page-5-5"></span>[4] J. Dierkes, [4.1 Planung, Beschreibung und Dokumentation von](https://doi.org/10.1515/9783110657807-018) [Forschungsdaten.](https://doi.org/10.1515/9783110657807-018) In *Praxishandbuch Forschungsdatenmanagement*, pp. 303–325, De Gruyter Saur, Berlin, Boston (2021)
- <span id="page-5-13"></span>[5] J. Fehr, J. Heiland, C. Himpe and J. Saak, [Best practices for replica](https://doi.org/10.3934/math.2016.3.261)[bility, reproducibility and reusability of computer-based experiments](https://doi.org/10.3934/math.2016.3.261) [exemplified by model reduction software.](https://doi.org/10.3934/math.2016.3.261) *AIMS Mathematics* 1, 261–281 (2016)
- <span id="page-5-15"></span>[6] J. Fehr, C. Himpe, S. Rave and J. Saak, [Sustainable research software](https://doi.org/10.5334/jors.307) [hand-over](https://doi.org/10.5334/jors.307). *Journal of Open Research Software* 9, article no. 5 (2021)
- <span id="page-5-18"></span>[7] P. B. Heidorn, [Shedding light on the dark data in the long tail of](https://doi.org/10.1353/lib.0.0036) [science.](https://doi.org/10.1353/lib.0.0036) *Library Trends* 57, 280–299 (2008)
- <span id="page-5-8"></span>[8] K. Hulek, F. Müller, M. Schubotz and O. Teschke, [Mathematical](https://doi.org/10.4171/news/113/14) [research data – an analysis through zbMATH references](https://doi.org/10.4171/news/113/14). *EMS Newsl.* 113, 54–57 (2019)
- <span id="page-5-6"></span>[9] M. Kindling and P. Schirmbacher, ["Die digitale Forschungswelt"](https://doi.org/10.1515/iwp-2013-0017) [als Gegenstand der Forschung / Research on digital research /](https://doi.org/10.1515/iwp-2013-0017) [Recherche dans la domaine de la recherche numérique](https://doi.org/10.1515/iwp-2013-0017). *Information – Wissenschaft & Praxis* 64, 127–136 (2013)
- <span id="page-5-9"></span>[10] T. Koprucki, K. Tabelow and I. Kleinod, [Mathematical research data.](https://doi.org/10.1002/pamm.201610458) *PAMM. Proc. Appl. Math. Mech.* 16, 959–960 (2016)
- <span id="page-5-12"></span>[11] M. Kostre, V. Sunkara, C. Schütte and N. D. Conrad, [Understanding](https://doi.org/10.1007/s41109-022-00492-w) [the romanization spreading on historical interregional networks in](https://doi.org/10.1007/s41109-022-00492-w) [Northern Tunisia](https://doi.org/10.1007/s41109-022-00492-w). *Applied Network Science* 7, article no. 53 (2022)
- <span id="page-5-0"></span>[12] M. S. Krafczyk, A. Shi, A. Bhaskar, D. Marinov and V. Stodden, [Learning from reproducing computational results: introducing three](https://doi.org/10.1098/rsta.2020.0069) principles and the *[Reproduction Package](https://doi.org/10.1098/rsta.2020.0069)*. *Philos. Trans. Roy. Soc. A* 379, article no. 20200069 (2021)
- <span id="page-5-1"></span>[13] K. Lejaeghere, G. Bihlmayer, T. Björkman, P. Blaha, S. Blügel, V. Blum, D. Caliste, I. E. Castelli, S. J. Clark, A. Dal Corso, S. de Gironcoli, T. Deutsch, J. K. Dewhurst, I. Di Marco, C. Draxl, M. Dułak, O. Eriksson, J. A. Flores-Livas, K. F. Garrity, L. Genovese, P. Giannozzi, M. Giantomassi, S. Goedecker, X. Gonze, O. Grånäs, E. K. U. Gross, A. Gulans, F. Gygi, D. R. Hamann, P. J. Hasnip, N. A. W. Holzwarth, D. Iuşan, D. B. Jochym, F. Jollet, D. Jones, G. Kresse, K. Koepernik, E. Küçükbenli, Y. O. Kvashnin, I. L. M. Locht, S. Lubeck, M. Marsman, N. Marzari, U. Nitzsche, L. Nordström, T. Ozaki, L. Paulatto, C. J. Pickard, W. Poelmans, M. I. J. Probert, K. Refson, M. Richter, G.-M. Rignanese, S. Saha, M. Scheffler, M. Schlipf, K. Schwarz, S. Sharma, F. Tavazza, P. Thunström, A. Tkatchenko, M. Torrent, D. Vanderbilt, M. J. van Setten, V. Van Speybroeck, J. M. Wills, J. R. Yates, G.-X. Zhang and S. Cottenier, [Reproducibility in density functional theory](https://doi.org/10.1126/science.aad3000) [calculations of solids](https://doi.org/10.1126/science.aad3000). *Science* 351, article no. aad3000 (2016)
- <span id="page-5-16"></span>[14] F. Matúš, [Conditional independences among four random](https://dx.doi.org/10.1017/S0963548300001747) [variables. II](https://dx.doi.org/10.1017/S0963548300001747). *Combin. Probab. Comput.* 4, 407–417 (1995)
- [15] F. Matúš, [Conditional independences among four random vari](https://dx.doi.org/10.1017/S0963548399003740)[ables. III. Final conclusion.](https://dx.doi.org/10.1017/S0963548399003740) *Combin. Probab. Comput.* 8, 269–276 (1999)
- <span id="page-5-17"></span>[16] F. Matúš and M. Studený, [Conditional independences among four](https://dx.doi.org/10.1017/S0963548300001644) [random variables. I.](https://dx.doi.org/10.1017/S0963548300001644) *Combin. Probab. Comput.* 4, 269–278 (1995)
- <span id="page-5-4"></span>[17] W. K. Michener, [Ten simple rules for creating a good data manage](https://doi.org/10.1371/journal.pcbi.1004525)[ment plan.](https://doi.org/10.1371/journal.pcbi.1004525) *PLOS Computational Biology* 11, article no. e1004525 (2015)
- <span id="page-5-2"></span>[18] C. Riedel, H. Geßner, A. Seegebrecht, S. I. Ayon, S. H. Chowdhury, R. Engbert and U. Lucke, [Including data management in research](https://dx.doi.org/10.18420/inf2022_114) [culture increases the reproducibility of scientific results.](https://dx.doi.org/10.18420/inf2022_114) In *Proceedings of INFORMATIK 2022*, Lecture Notes in Informatik P-326, pp. 1341–1352, Gesellschaft für Informatik, Bonn (2022)
- <span id="page-5-3"></span>[19] M. Schappals, A. Mecklenfeld, L. Kröger, V. Botan, A. Köster, S. Stephan, E. J. García, G. Rutkai, G. Raabe, P. Klein, K. Leonhard, C. W. Glass, J. Lenhard, J. Vrabec and H. Hasse, [Round robin study:](https://doi.org/10.1021/acs.jctc.7b00489) [Molecular simulation of thermodynamic properties from models](https://doi.org/10.1021/acs.jctc.7b00489) [with internal degrees of freedom](https://doi.org/10.1021/acs.jctc.7b00489). *J. Chem. Theory Comput.* 13, 4270–4280 (2017)
- <span id="page-5-11"></span>[20] D. Schober, B. Smith, S. E. Lewis, W. Kusnierczyk, J. Lomax, C. Mungall, C. F. Taylor, P. Rocca-Serra and S.-A. Sansone, [Survey](https://doi.org/10.1186/1471-2105-10-125)[based naming conventions for use in OBO Foundry ontology](https://doi.org/10.1186/1471-2105-10-125) [development.](https://doi.org/10.1186/1471-2105-10-125) *BMC Bioinformatics* 10, article no. 125 (2009)
- <span id="page-6-4"></span>[21] P. Šimeček, A short note on discrete representability of independence models. In *Proceedings of the 3rd European Workshop on Probabilistic Graphical Models*, pp. 287–292, Action M Agency, Prague (2006)
- <span id="page-6-1"></span>[22] O. Teschke, [Some heuristics about the ecosystem of mathematics](https://doi.org/10.1002/pamm.201610460) [research data](https://doi.org/10.1002/pamm.201610460). *PAMM. Proc. Appl. Math. Mech.* 16, 963–964 (2016)
- <span id="page-6-3"></span>[23] The LMFDB Collaboration, The L-functions and modular forms database. <http://www.lmfdb.org> (2022)
- <span id="page-6-2"></span>[24] The MaRDI consortium, *[MaRDI: Mathematical Research Data](https://doi.org/10.5281/zenodo.6552436) [Initiative proposal](https://doi.org/10.5281/zenodo.6552436)*. Zenodo (2022)
- <span id="page-6-0"></span>[25] M. Wilkinson, M. Dumontier, IJ. J. Aalbersberg, G. Appleton, M. Axton, A. Baak, N. Blomberg, J.-W. Boiten, L. O. Bonino da Silva Santos, P. Bourne, J. Bouwman, A. Brookes, T. Clark, M. Crosas, I. Dillo, O. Dumon, S. Edmunds, C. Evelo, R. Finkers and B. Mons, [The FAIR Guiding Principles for scientific data management and](https://doi.org/10.1038/sdata.2016.18) [stewardship](https://doi.org/10.1038/sdata.2016.18). *Scientific Data* 3, article no. 160018 (2016)

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