

ERME column

regularly presented by Frode Rønning and Andreas Stylianides

In this issue, with a brief report from CERME13 and a presentation of the Thematic Working Group on Mathematics Teaching and Teacher Practice(s), TWG19, by the group leaders Reidar Mosvold, Mark Hoover, Helena Grundén, Siún Nic Mhuirí and Chrysoula Choutou

CERME13

The 13th Congress of the European Society for Research in Mathematics Education, CERME13, was held at the Eötvös Loránd University, Budapest, Hungary, on 10–14 July 2023.¹ The CERMEs are usually held in February, but due to uncertainties regarding the Covid situation at the time of decision, it was decided to hold the congress in the summer.

A total of 941 participants from 54 countries attended CERME13, distributed over 27 Thematic Working Groups (TWGs). A record 616 papers and 141 posters were accepted and presented. Two plenary lectures were delivered: Berta Barquero from the University of Barcelona, Spain, gave a lecture titled *Mathematical modelling as a research field: Transposition challenges and future directions*, and László Lovász from Hungary, an Abel Prize laureate,² gave a lecture with the title *Why is mathematics beautiful?* Finally, a plenary panel around the topic of *Bridging the research-practice gap*, was chaired by João Pedro da Ponte (Portugal).

The first CERME was held 35 years ago, in 1998, and attracted 120 participants from 24 countries. The growth in the number of participants shows that CERME enjoys wide attraction, both in Europe and beyond, but it has now come to a point where further growth is neither feasible nor desirable.

From CERME14, the congress will be back on track as regards time of the year, as it will be held on 4–8 February 2025, at the Free University of Bozen-Bolzano, Italy. This university also arranged the virtual CERME12 in 2022 and will now have the opportunity to host a real, on-site conference.

CERME Thematic Working Groups

We continue the initiative of introducing the CERME Thematic Working Groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for people working

in pure and applied mathematics. Our aim is to disseminate developments in mathematics education research discussed at CERMEs and enrich the ERME community with new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.

CERME Thematic Working Group 19: Mathematics Teaching and Teacher Practice(s)

Introduction

Research on mathematics teaching, whether in elementary schools, high schools, or universities, has a long history, but the emphasis in research on teaching has shifted over the decades. Often the focus has been on teachers, classrooms, and their characteristics, rather than on teaching as an activity itself. When teaching has been in focus, it has been thought about in different ways. In the first *Handbook of Research on Teaching*, Henderson [3, p. 1007] described teaching in mathematical terms: “ x teaches y to z ,” and he proposed that teacher behavior – what teachers do – was the most significant factor for research on mathematics teaching. Teacher behavior could, and perhaps should, focus on the interactions between teacher (x) and students (z) around some content (y), but early research on teaching developed an emphasis on simple, measurable behaviors with little regard for content or reflective practice.

Then, after several decades of research on teacher behavior, Shulman [4] claimed that the role of content had become a missing paradigm in research on teaching. Following Shulman, researchers in the field shifted attention to content knowledge for teaching mathematics. This shift resulted in studies that attempted to understand and measure *teachers’ knowledge*. Some studies of teachers’ knowledge draw on a cognitive perspective, highlighting teachers’ thinking and cognition, whereas other studies emphasize what mathematical knowledge teachers need, or ought to have. However, Ball and colleagues offer a practice-based focus on what it means to know and do mathematics in and for teaching [1]. Such a shift requires considering teaching as work to be done, where this work involves certain entailments or demands that teachers

¹ <https://cerme13.renyi.hu>

² <https://abelprize.no/abel-prize-laureates/2021>

face. More recently, there has been increased focus on the social, cultural, and political aspects of mathematics teaching. Aguirre et al. [2] argue that some research has moved beyond sociocultural views and suggests a sociopolitical turn, which means that issues such as equity, identity and power are at play in research on mathematics teaching and learning.

Across this historical development, three underlying conceptions of teaching can be identified in research on teaching. Mathematics teaching can be considered as:

- What teachers *do*.
- What teachers *should* or *ought* to do (or know).
- *Work to be done*.

Thematic Working Group (TWG) 19 seeks to navigate these trends and underlying conceptions in efforts to study mathematics teaching, from preschool to graduate school.

History of the TWG

Ever since the first CERME conference in 1998, our TWG was part of a group that was called *From a Study of Teaching Practices to Issues in Teacher Education*. This group existed for several years, before it split up into three groups at CERME8. One of the new groups emphasized teacher education and professional development, and another group targeted teacher knowledge, beliefs, and identity. TWG19 was initially called *Mathematics Teacher and Classroom Practices*, but after CERME10, the name was changed to *Mathematics Teaching and Teacher Practice(s)*. This change of name was meant to signal that the primary emphasis was on *teaching*, and not on *teachers*. It was also meant to clarify that the emphasis was on *teacher practice(s)*, and not on any practices that might occur in classrooms.

In tandem with this change of name, we recognized – and started to explicitly address – some key challenges. Unlike research on mathematical learning, research on mathematics teaching does not have any grand theory. Instead, researchers have developed diverse frameworks, conceptions, theories, and methods – many of which are left implicit. Managing this diversity has been challenging. As a community, we felt a need to develop common frames or conceptions, or at least find ways to more explicitly discuss the diversity. Without shared language and meanings, it was difficult to understand each other's research.

Current efforts

In the past few conferences, TWG19 has initiated responses to these challenges. First, we have made efforts to collaboratively explore the meaning of teaching. We have asked participants to be explicit about what they mean by “teaching” in their papers, we have facilitated discussions of the meanings of teaching across papers at the conferences, and we have explored distinctions in the meaning of teaching in different studies. Second, we have

proposed four analytic domains as a way of identifying the primary focus in studies on mathematics teaching. Participants have been encouraged to consider their own work in relation to one of the following domains:

1. Consideration of mathematics and the central endeavor of extending the subject to students.
2. Becoming acquainted with, relating to, and responding to students as people and learners.
3. Organization and enactment of design, interaction, and discourse of teaching and learning.
4. Attending to broader social, cultural, and political issues that matter for teaching and learning, including imperatives of social justice.

Third, we have offered sets of shared data and invited participants to use these data sets in their papers and presentations. As a result, several papers have applied different lenses to analyze these common data sets, and all participants in the group have been given access to these data sets to provide a common ground for discussions.

Conclusion

Although the challenges in research on mathematics teaching have not been resolved, we have experienced TWG19 as a productive arena for discussing and developing research in this field. The four domains draw on a conception of teaching as instructional interactions between teachers and students around some mathematical content within some particular environment (adding an important contextual factor to the original set of variables proposed by Henderson). The five domains have helped us to be more explicit about what we mean by teaching and have stimulated discussion of how different conceptions communicate with each other. Additionally, the shared data initiative continues to provide a common ground for exploring and discussing mathematics teaching. In this work, the perspectives of mathematics and mathematicians continue to be important, as the mathematical issues at play, whether in teaching young children or adults, are surprisingly subtle and deep. We welcome broad engagement in the study of mathematics teaching. More than just encouraging it, we see it as essential to growing understanding and improving practice.

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Reidar Mosvold is a professor of mathematics education at the University of Stavanger. He enjoys being involved with research and development of mathematics teaching. His research interests focus on the work of leading mathematical discussions, what it means to be smart in mathematics, and methods for studying mathematics teaching.

reidar.mosvold@uis.no

Mark Hoover is a research scientist at the University of Michigan. He is concerned with personal and professional relationships in the presence of power. His research investigates the work entailed in mathematics teaching, extending to teaching and teacher education across subject areas, the subject-matter work of teaching, and conceptual-analytic methods for studying teaching.

mhoover@umich.edu

Helena Grundén is deputy head of school, teacher educator, and researcher at the Dalarna University in Sweden. Her research interests include planning in mathematics teaching as well as social and structural issues that relate to mathematics education.

hgn@du.se

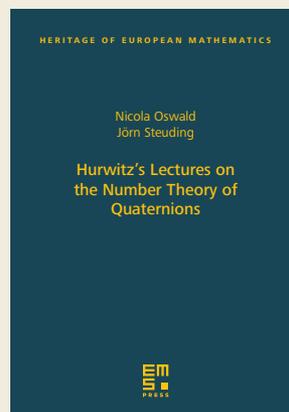
Siún Nic Mhuirí is an assistant professor of mathematics education in the Institute of Education, Dublin City University. She is interested in the agency and authority of teachers and students. Her research investigates the role of talk in classroom practice, and the design of initial and continuing teacher education.

siun.nicmhuiri@dcu.ie

Chrysoula Choutou holds a PhD in mathematics education from the National and Kapodistrian University of Athens. Her research centers on investigating the connections between mathematics and visual arts in teaching and learning, and how secondary school mathematics teachers can integrate visual arts into their teaching through collaboration with visual art teachers.

chrychou@math.uoa.gr

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