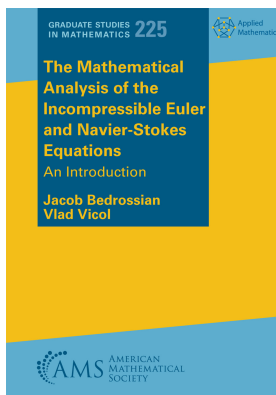


Book review

The Mathematical Analysis of the Incompressible Euler and Navier–Stokes Equations: An Introduction by Jacob Bedrossian and Vlad Vicol

Reviewed by Hao Jia



The book under review is an introduction to the mathematical theory of incompressible Navier–Stokes and Euler equations, with an emphasis on the regularity theory of solutions, and includes one chapter on recent research trends and important achievements. The authors, who are leading figures in the analysis of partial differential equation of fluids, have made important contributions in subjects like hydrodynamic stability at high Reynolds

numbers, uniqueness of weak solutions to Navier–Stokes equations, statistical laws in fluid equations, shock formation in compressible fluids, stability of coherent solutions for Euler equations, among many other accomplishments.

The main topics covered include physical derivation and interpretation of key quantities that appear in the equations, local well-posedness and regularity criteria of classical solutions, existence theory of mild solutions, existence and partial regularity of Leray–Hopf weak solutions, and a survey of selected research topics. The appendix provides essential technical tools used throughout the book. Each chapter also contains a list of exercises which complement and extend the contents in that chapter. The book is essentially self-contained, with complete proofs of central theorems. The survey discusses important research directions not covered in the previous chapters, such as stationary solutions, self-similar solutions, potential singularity formation scenarios, hydrodynamic stability problems and Onsager’s conjectures, and more. Many of these directions are still of great interest in the research community, to which the authors have made seminal contributions.

It is clear that this is not a comprehensive monograph on Euler and Navier–Stokes equations. Instead, the authors are sharply focused on the main ideas and key techniques in the simplest and most important cases. For example, the existence theory is presented in either the whole space or periodic domains, thus avoiding technical issues related to nontrivial boundaries. In the reviewer’s opinion, for an introductory text aimed primarily at graduate students and mathematicians who have a general interest in equations of fluid dynamics, this is an excellent choice. It allowed the authors to present the core ideas in a direct and concise way. It also made it possible to cover both the incompressible Euler and Navier–Stokes equations, which is an important feature of the book. As a result, the physically significant issue of inviscid limit from the Navier–Stokes to Euler equations can be brought up naturally.

An extremely valuable contribution of the book, one that will be much appreciated by readers, is the survey of research topics in Chapter 6, where the authors provide a general review of twelve current research directions concerned with the equations of incompressible fluids. The surveys are not just compilations of references. Rather, the motivation, history, known results and open problems are all expertly presented with comments on promising future directions. This chapter will be useful not only for graduate students, but also for researchers, providing a broad view of the research landscape in incompressible fluid equations and a source of inspiration for new projects.

In summary, this is an excellent introductory textbook suitable for graduate students, instructors and general mathematicians with an interest in equations of fluids. Researchers on Navier–Stokes and Euler equations will also find it handy to keep in their library for quick references. By design, the book is *lightweight* and *focused*, which makes it a fantastic choice for a one-semester course on the subject. The readers will benefit from the clear presentation of the general material and deep insights in the survey, while having an enjoyable learning experience.

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Hao Jia is an associate professor of mathematics at the University of Minnesota. His main current research interest is partial differential equations arising from the mathematical theory on the dynamics of incompressible fluid flows. He is on the editorial board of the journals Communications on Pure and Applied Analysis and Journal of Differential Equations.

jia@umn.edu

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