Thematic Working Group on Teaching and Learning of Discrete and Computational Mathematics, TWG11

ERME column regularly presented by Frode Rønning and Andreas Stylianides

In this issue presented by the group leaders Simon Modeste, Sylvia van Borkulo, Ulrich Kortenkamp, Janka Medová and David Zenkl

CERME Thematic Working Groups

We continue the initiative of introducing the CERME Thematic Working Groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for people working in pure and applied mathematics. Our aim is to disseminate developments in mathematics education research discussed at CERMEs and enrich the ERME community with new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.

Introducing CERME's Thematic Working Group 11 – Teaching and Learning of Discrete and Computational Mathematics

This TWG was newly created for CERME14, emerging from the encounter of the interests of a previously existing group entitled *Algorithms* [3] and a prospective group project on discrete mathematics, based on common observations of recent trends in mathematics linked with computer science.

Indeed, the development of computer science in the last decades has generated changes in the contents and practices in mathematics, both in the advanced and theoretical dimensions and in the very applied and practical aspects. In particular, it has contributed to the recent emphasis and success of discrete mathematics, a classical topic in mathematics. Also, the development of computer science has led to the growth of computational approaches in mathematics, fostering applications and new ways of thinking and doing mathematics. These evolutions in the scientific field of mathematics, and the related needs for future generations, require and produce new contents in school that our TWG intends to address. The topics covered by the TWG take a growing place in many current curricula around the world and entered the common core of mathematics as described by international organizations like OECD. For this reason, research developments and insights are expected by teachers, teacher educators, and policymakers. Expert mathematicians and computer scientists have an important role to play, in collaboration with educators, to better understand the developments related to these trends and think about the changes in current and future curricula.

To address these challenges at CERME14, the TWG invited research papers discussing empirical, theoretical, methodological, or philosophical issues pertaining to the teaching and learning of discrete and computational mathematics, including the following themes: all aspects of discrete mathematics, i.e., both classical subjects, and subjects invoking new needs for computer science, namely, combinatorics, graph theory, discrete geometry, automata, game theory, cryptography, etc.; manipulation and representation of mathematical objects and data in computer science, simulations and discrete modelling, new computational views on classical mathematical objects, computational approaches in mathematics; links between mathematics and computer science, their foundations, concepts, ideas, and methods; algorithms in mathematics, the study and analysis of algorithms; computational thinking, and programming in mathematics; proof, proving, and problem solving in discrete and computational mathematics. Given the current curricular issues, our TWG has interest in these topics in pre-school, primary, and secondary levels, as well as studies at university level, and the implications for teacher education.

We were pleased that the 25 accepted papers and posters presented at CERME14 covered a wide diversity of the targeted topics. We organized them in four themes. The first theme was "Implementing computational thinking in mathematics." Computational thinking, although having various and sometimes blurry definitions [1], appears more and more in curricula [4], and focuses on the development of computational procedures and automation of mathematical concepts. This was explored at different levels and regarding various contents of mathematics, like geometry, functions, algebra, and also in programming for mathematics, and as a skill in itself that students must develop. The second theme was "Links between mathematics and informatics." In this theme, theoretical and experimental research contributed to discussing the relation between the two disciplines, including topics like the use of automatic proof checkers, algorithmics and programming in mathematical problem solving, the differences and similarities between mathematics and computer science activities and ways of thinking, and what concepts computer science

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can bring to think about mathematics education issues. The third and fourth themes were connected to discrete mathematics [2]. The third theme explored the "nature and diversity of discrete mathematics," questioning specific topics like graph theory, or tiling problems, but also the way discrete mathematics can connect or is connected with other mathematical topics. The fourth theme was covered by a group of communications specifically dedicated to "combinatorics," which seems to be the most developed part of discrete mathematics in school curricula in many countries. Combinatorics, more traditional in curricula, has been much investigated in the past, and contributions covered various questions in this area, such as students' recurrent difficulties and errors, specific strategies and approaches in combinatorics (like identifying "isomorphisms" between combinatorial structures in counting problems), and teaching and learning issues, including the use of programming/algorithmic skills in combinatorics.

Only one theme was underrepresented in the TWG: teacher education issues. This can be partly explained by the novelty of the matter addressed, which only starts to appear in the curricula, and although integrated in teacher education programmes, may not be ready for research and for the early stage of the TWG, which met for the first time at CERME14. We have also identified some relevant contributions that were submitted to other TWGs, which means that we should try to attract such contributions in the future. In any case, developing this dimension on teacher education is an identified goal for the next CERME. It became clear from the discussion that immediate actions that could affect teacher education must be developed, and that research is needed to support and develop it.

Apart from that, we were very happy to see that in the discussions, participants, whatever their principal interest, were equally involved in our two main topics, computational mathematics and discrete mathematics. The collective discussions and work showed rich connections between discrete and computational mathematics and confirmed that it is interesting to join and make them interact. Many perspectives have been opened, and we hope that the TWG will be able to develop long-term collaborations on these "hot topics" for mathematics education.

References

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