

Preface

The origin of this book lies at the beginning of my graduate studies, when I just could not understand Uhlenbeck compactness, let alone see whether it would also hold for my cases – on manifolds with boundary. There seemed to be certain gaps between standard mathematics education and the analytic background needed to understand a very fundamental research paper in Yang-Mills theory.

A first gap was closed while I was working with Laurent Lazzarini – we finally understood the L^p -estimate for the operator $d \oplus d^*$. From there I went on through the depth of Uhlenbeck compactness - guided and encouraged by my supervisor, Dietmar Salamon, and always keeping my boundaries in mind. I had to overcome some obstacles, found a few subtleties, and finally arrived at a detailed understanding of Uhlenbeck compactness and its (not far from obvious) generalizations to manifolds with boundaries and manifolds exhausted by compact sets. All this work seemed to be worth writing down, so I ended up writing the book that I would have needed at the beginning of my graduate studies: A selfcontained exposition of Uhlenbeck compactness with all the analytic details, which only refers back to standard textbooks for classical results.

After having difficulties in finding references on L^p -results for the Neumann boundary value problem I included a preliminary part on that topic. This provides the required results for the Neumann problem and also shows the general philosophy behind elliptic boundary value problems – it taught me the deep truth in Uhlenbeck’s sentence “Elliptic systems are well-behaved on Sobolev spaces”.

So this book intends to be a guide to students on their way into the analysis of Yang-Mills theory. I also hope that it will be a useful reference for those who need to know about a particular detail in or behind Uhlenbeck compactness.

Let me stress that I do not claim any original work. The minor generalizations of Uhlenbeck compactness that are stated in this book have been known before. However, there are a lot of details and alternative approaches to certain parts of the proofs that probably cannot be found elsewhere. These bits and pieces are specified in the introduction.

Much credit for this book goes to Dietmar Salamon - for the idea as a start, for all the help with obstacles, but also for teaching me how to overcome them myself and finally, how to write mathematics. I’m also glad to have this opportunity to thank the ‘symplectic gang’ in and around the ETH Zürich for a great working atmosphere, stimulating discussions, and some glorious symplectic action!