Preface

The subject of Fukaya categories has a reputation for being hard to approach. This is due to the amount of background knowledge required (taken from homological algebra, symplectic geometry, and geometric analysis), and equally to the rather complicated nature of the basic definitions. The present book is intended as a resource for graduate students and researchers who would like to learn about Fukaya categories, and possibly use them in their own work. I have tried to focus on a rather basic subset of topics, and to describe these as precisely as I could, filling in gaps found in some of the early references. This makes for a rather austere style (for that reason, a thorough study of this book should probably be complemented by reading some of the papers dealing with applications). A second aim was to give an account of some previously unpublished results, which connect Fukaya categories to the theory of Lefschetz fibrations. This becomes predominant in the last sections, where the text gradually turns into a research monograph.

I have borrowed liberally from the work of many people, first and foremost among them Fukaya, Kontsevich, and Donaldson. Fukaya's foundational contribution, of course, was to introduce A_∞-structures into symplectic geometry. On the algebraic side, he pioneered the use of the A_∞-version of the Yoneda embedding, which we adopt systematically. Besides that, several geometric ideas, such as the role of Pin structures, and the construction of A_∞-homomorphisms in terms of parametrized moduli spaces, are taken from the work of Fukaya, Oh, Ohta and Ono. Kontsevich introduced derived categories of A_∞-categories, and is responsible for much of their theory, in particular the intrinsic characterization of exact triangles. He also conjectured the relation between Dehn twist and twist functors, which is one of our main results. Finally, in joint work with Barannikov, he suggested a construction of Fukaya categories for Lefschetz fibrations; we use a superficially different, but presumably equivalent, definition. Donaldson's influence is equally pervasive. Besides his groundbreaking work on Lefschetz pencils, he introduced matching cycles, and proposed them as the starting point for a combinatorial formula for Floer cohomology, which is indeed partly realized here. Other mathematicians have also made important contributions. For instance, parts of our presentation of Picard–Lefschetz theory reflect Auroux' point of view. A result of Smith, namely that the vanishing cycles in a four-dimensional Lefschetz pencil necessarily fill out the fibre, was crucial in suggesting that such cycles might "split-generate" the Fukaya category. Besides that, work of Fukaya–Smith on cotangent bundles provided a good testing-ground for some of the more adventurous ideas about Lefschetz fibrations. Our approach to transversality issues is the result of several conversations with Lazzarini. Finally, Abouzaid's suggestions greatly improved the discussion of symplectic embeddings.

Among organizations, I would like to thank ETH Zürich for inviting me to teach the course on which this book is based; the NSF for financial support (under grants DMS-9729992 and DMS-0405516); and the EMS Publishing House for accepting the manuscript, and shepherding it to publication.