

Contents

0	Introduction	vii
1	Function spaces	1
	1.1 Function spaces for functions of x	1
	1.2 Functions of t and x	3
2	The deterministic 2D Navier-Stokes Equation	5
	2.1 Leray decomposition	5
	2.2 Properties of the nonlinearity \mathbf{B}	8
	2.3 The existence and uniqueness theorem	10
	2.4 Improving the smoothness of solutions	14
	2.5 The NS semigroup	18
	2.6 Singular forces	19
	2.7 Some hydrodynamical terminology	22
3	Random kick-forces	24
	3.1 Ingredients for the constructions	24
	3.2 The kicked NSE	25
	3.3 Stationary measures	27
	3.4 More estimates	28
4	White-forced equations	30
	4.1 White in time forces	30
	4.2 The white-forced 2D NSE	31
	4.3 Estimates for solutions	33
	4.4 Stationary measures	36
	4.5 High-frequency random kicks	37
5	Preliminaries from measure theory	39
	5.1 Weak convergence of measures and Lipschitz-dual distance	39
	5.2 Variational distance	40
	5.3 Coupling	41
	5.4 Kantorovich functionals	42
6	Uniqueness of a stationary measure: kick-forces	43
	6.1 The main lemma	43
	6.2 Weak solution of (6.1)	45
	6.3 The theorem	46
	6.4 Corollaries from the theorem	50
	6.5 3D NSE with small random kicks	51
	6.6 Stationary measures and random attractors	52
	6.7 Appendix: Summary of the proof of Theorem 6.4	53

7	Uniqueness of a stationary measure: white-forces	56
	7.1 The main theorem	56
	7.2 Stationary measures for equation, perturbed by high frequency kicks	58
8	Ergodicity and the strong law of large numbers	60
9	The martingale approximation and CLT	63
10	The Eulerian limit	66
	10.1 White-forces, proportional to the square-root of the viscosity	66
	10.2 One negative result	71
	10.3 Other scalings	73
	10.4 Discussion	74
	10.5 Kicked equations	75
11	Balance relations for the white-forced NSE	77
	11.1 The balance relations	77
	11.2 The co-area form of the balance relations	80
12	Comments	83
	Bibliography	88
	Index	93