Preface

This book introduces the reader to the theory of locally compact groups, leading from the basics about topological groups to more involved topics, including transformation groups, the Haar integral, and Pontryagin duality. I have also included several applications to the structure theory of locally compact Abelian groups, to topological rings and fields. The presentation is rounded off by a chapter on topological semigroups, paying special respect to results that identify topological groups inside this wider class. In order to show the results from Pontryagin theory at work, I have also included the determination of those locally compact Abelian groups that are homogeneous in the sense that their automorphism group acts transitively on the set of non-trivial elements. A crucial but deep tool for any deeper understanding of locally compact groups is the approximation by Lie groups. The chapter on Hilbert's fifth problem gives an overview. The chart following this preface gives a rough impression of the logical dependence between the sections.

During my academic career, I have repeatedly lectured on topics from topological algebra. Apart from a regular seminar including topics from the field, I have given graduate courses on topological groups (1994/95), locally compact groups (1995/96), Pontryagin duality (1996/97), Haar measure (1997), and topological algebra (1999/2000). The present notes reflect the topics treated in these courses. A suitable choice from the material at hand may cover one-semester courses on topological groups (Sections 3, 4, 5, 6, 7, 8, 9, 10, 11), locally compact Abelian groups (Sections 3, 4, 6, 12, 14, 20, 21, 22, 23, 24), topological algebra (groups, rings, fields, semigroups: Sections 3, 4, 5, 6, 9, 10, 11, 26, 28, 29, 30, 31).

I have tried to keep these notes essentially self-contained. Of course, as with any advanced topic, there are limits. A reader is supposed to have mastered linear algebra, but only a basic acquaintance with groups is required. Fundamental topological notions (topologies, continuity, neighborhood bases, separation, compactness, connectedness, filterbases) are treated in Section 1 and in Section 2. The more advanced topic of dimension is included only as a reference for the outline in Chapter H. The section about Haar integral draws (naturally) from functional analytic sources.

Almost every section (with the systematic exception of those in Chapter H) is accompanied by exercises. These have been tested in class, but of course this is no guarantee that they will work well again with any other group of students. Every reader is advised to use the exercises as a means to check her understanding of the topics treated in the text. Occasionally, the exercises also provide further examples.

A remark on the bibliography is in order. The present book is meant as a student text, and historic comments are kept to a minimum. We give references to the literature where we need results or techniques that are beyond the scope of this book. Suggestions for further reading would surely include the two volumes by E. Hewitt and K. A. Ross [15], [16]. Quite recent contributions are the impressive monographs by K. H. Hofmann and S. A. Morris [23], [24]. About locally compact abelian groups, we only mention the book by D. L. Armacost [2]. The notes by I. Kaplansky [35] treat abelian groups but go well beyond into the solution of Hilbert's Fifth Problem. Topological fields are the subject of the monographs by N. Shell [57], S. Warner [65], W. Więsław [68], and (under the pretext of doing number theory) the one by A. Weil [67]. The theory of locally compact groups naturally incorporates deep results from Lie theory. Among the many books about that subject, we mention the ones by N. Bourbaki [4], S. Helgason [14], G. Hochschild [18], A. L. Onishchik and E. B. Vinberg [43], and V. S. Varadarajan [64].

The abstract notion of a topological group seems to appear first in a paper by F. Leja [38]. Historical hallmarks of the theory are the books by L. S. Pontrjagin [48] and A. Weil [66].

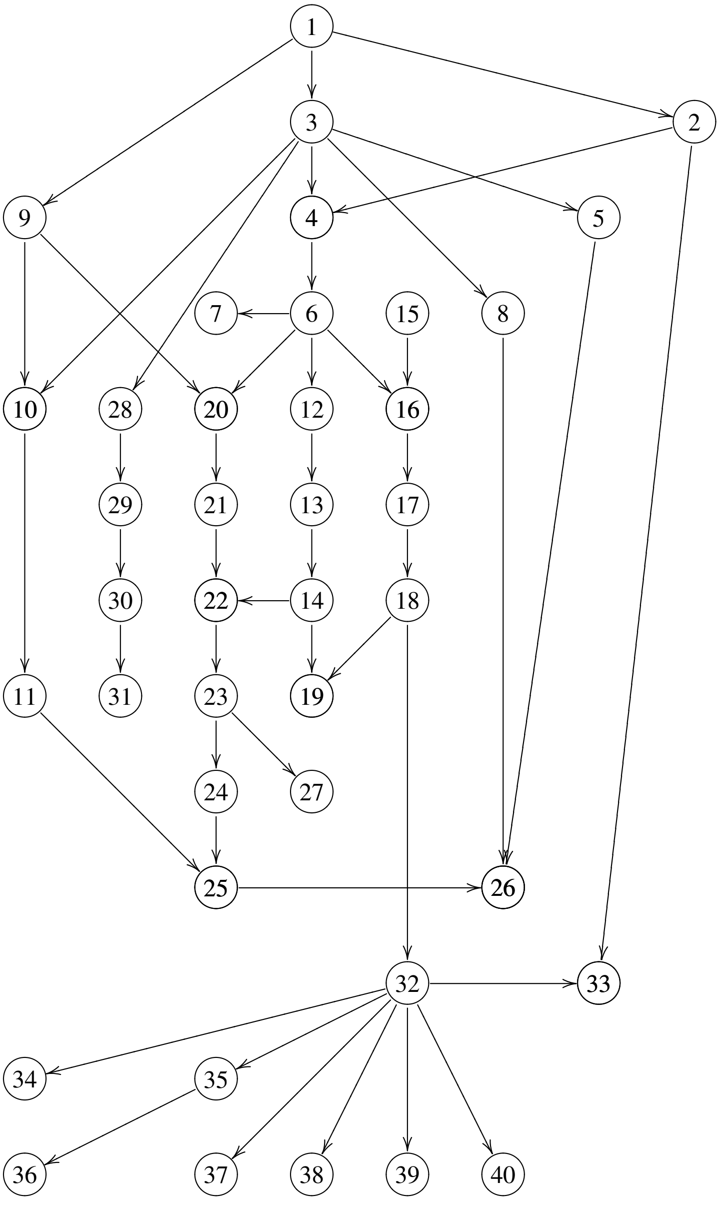
I was introduced to the theory of topological groups in a course at the university at Tübingen, given by H. Reiner Salzmann, during the summer term in 1987. That course started with an introduction to the basics (including subgroups, quotients, separation properties, and connectedness), general properties of locally compact groups (existence of open subgroups, extension properties), a discussion of topological transformation groups (leading to Freudenthal's results about locally compact orbits of locally compact Lindelöf groups). The main part of that course consisted of a discussion of Pontryagin duality, its proof and several consequences, culminating in the classification of compact Abelian groups. Surely, these lectures contributed to my decision to take up research in mathematics. What impressed me deeply was the interplay of subtly interwoven theories (topology, group theory, functional analysis), leading to deep results, with applications in pure as well as applied mathematics. Later on, I had the opportunity to work with Karl Heinrich Hofmann at Darmstadt. During this lasting collaboration, the seed of fascination with the topics was cultivated, ripening into a full-grown addiction to the theory of locally compact groups and its various applications.

In my own teaching, I try to pass on the beauty of the subject as well as the fascination that my academic teachers have instilled in me. I do hope that these notes help to advance this fascination.

Many students and colleagues have read and criticized scripts accompanying my lecture courses, and parts of the present version. Explicitly, I wish to thank Andrea Blunck, Martin Bulach, Agnes Diller, Helge Glöckner, Jochen Hoheisel, Martin Klausch, Peter Lietz, and Bernhild Stroppel. The errors that remain are mine.

Stuttgart, December 2005 Markus Stroppel

Logical Dependence between the Sections



An arrow from k to ℓ in this chart indicates that Section k contains material that is necessary for Section ℓ.