

IRMA Lectures in Mathematics and Theoretical Physics 6

Edited by Christian Kassel and Vladimir G. Turaev

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Athanase Papadopoulos

Metric Spaces, Convexity and Non-positive Curvature

Second edition



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