

## Introduction

This book consists of lecture notes for a course given at the EMS Summer School on Noncommutative Geometry and Applications, at Monsaraz and Lisboa, Portugal in September, 1997. These were made available in preprint form on the ArXiv, as physics/9709045, at that time. In updating them for publication, I have kept to the original plan, but have added citations of more recent papers throughout. An extra final chapter summarizes some of the developments in noncommutative geometry in the intervening years.

The course sought to address a mixed audience of students and young researchers, both mathematicians and physicists, and to provide a gateway to noncommutative geometry, as it then stood. It already occupied a wide-ranging area of mathematics, and had received some scrutiny from particle physicists. Shortly thereafter, links to string theory were found, and its interest for theoretical physicists is now indisputable.

Many approaches can be taken to introducing noncommutative geometry. In these lectures, the focus is on the geometry of Riemannian spin manifolds and their noncommutative cousins, which are ‘spectral triples’ determined by a suitable generalization of the Dirac operator. These ‘spin geometries’, which are spectral triples with certain extra properties, underlie the noncommutative geometry approach to phenomenological particle models and recent attempts to place gravity and matter fields on the same geometrical footing.

The first two chapters are devoted to commutative geometry; we set up the general framework and then compute a simple example, the two-sphere, in noncommutative terms. The general definition of a spin geometry is then laid out and exemplified with the noncommutative torus. Enough details are given so that one can see clearly that noncommutative geometry is just ordinary geometry, extended by discarding the commutativity assumption on the coordinate algebra. Classification up to equivalence is dealt with briefly in Chapter 7.

Other chapters explore some of the tools of the trade: the noncommutative integral, the role of quantization, and the spectral action functional. Physical models are not treated directly (these were the subject of other lectures at the Summer School), but most of the mathematical issues needed for their understanding are dealt with here. The final chapter is a brief overview of the profusion of new examples and applications of noncommutative spaces and spectral triples.

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