Preface

This book grew out of two-semester courses given by the second-named author in 1996/97 and 1999/2000, and the first-named author in 2005/06. These lectures were directed to graduate students and PhD students having a working knowledge in calculus, measure theory and in basic elements of functional analysis (as usually covered by undergraduate courses).

It is one of the main aims of this book to develop at an accessible, moderate level an L_2 theory for elliptic differential operators of second order,

$$Au = -\sum_{j,k=1}^{n} a_{jk}(x)\frac{\partial^2 u}{\partial x_j \partial x_k} + \sum_{l=1}^{n} a_l(x)\frac{\partial u}{\partial x_l} + a(x)u, \qquad (*)$$

on bounded C^∞ domains Ω in $\mathbb{R}^n$, including a priori estimates for homogeneous and inhomogeneous boundary value problems in terms of (fractional) Sobolev spaces on Ω and on its boundary $\partial\Omega$, and a related spectral theory. This will be complemented by a few L_p assertions mostly connected with degenerate elliptic equations.

This book has 7 chapters. The first chapter deals with the well-known classical theory for the Laplace–Poisson equation and harmonic functions. It may also serve as an introduction to the typical questions related to boundary value problems. Chapter 2 collects the basic ingredients of the theory of distributions, including tempered distributions and Fourier transforms. In Chapters 3 and 4 we introduce Sobolev spaces on $\mathbb{R}^n$ and in domains, including embeddings, extensions and traces. The heart of the book is Chapter 5 where we develop an L_2 theory for elliptic operators of type $(*)$. Chapter 6 deals with some specific problems of the spectral theory in Hilbert spaces and Banach spaces on an abstract level, including approximation numbers, entropy numbers, and the Birman–Schwinger principle. This will be applied in Chapter 7 to elliptic operators of type $(*)$ and their degenerate perturbations. Finally we collect in Appendices A–D some basic material needed in the text, in particular some elements of operator theory in Hilbert spaces.

The book is addressed to graduate students and mathematicians seeking for an accessible introduction to some aspects of the theory of function spaces and its applications to elliptic equations. However it is not a comprehensive monograph, but it can be used (so we hope) for one-semester or two-semester courses (as we did). For that purpose we interspersed some *Exercises* throughout the text, especially in the first chapters. Furthermore each chapter ends with *Notes* where we collect some references and comments. We hint in these Notes also at some more advanced topics, mostly related to the recent theory of function spaces, and the corresponding literature. For this reason we collect in Appendix E a few relevant assertions.

In addition to the bibliography, there are corresponding indexes for (cited) *authors*, *notation*, and *subjects* at the end, as well as a *list of figures* and *selected solutions* of those exercises which are marked by a $*$ in the text. References are ordered by names, not by labels, which roughly coincides, but may occasionally cause minor deviations.

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