

Contents

| | |
|---|----|
| Preface | v |
| Preface to the Russian Edition | vi |
| 1 Introduction to finite group theory | 1 |
| 1 Main definitions | 1 |
| 2 Lagrange's theorem. Normal subgroups and factor groups | 4 |
| 3 Homomorphism theorems | 6 |
| 4 Cayley's theorem | 7 |
| 5 Double cosets | 9 |
| 6 Actions of groups on sets | 10 |
| 7 Normalizers and centralizers. The centers of finite p -groups | 12 |
| 8 Sylow's theorem | 13 |
| 9 Direct products of groups | 15 |
| 10 Finite simple groups | 16 |
| 11 The simplicity of the alternating group A_n for $n \geq 5$ | 18 |
| 12 A_5 as the rotation group of an icosahedron | 19 |
| 13 A_5 as the first noncyclic simple group | 20 |
| 14 A_5 as a projective special linear group | 22 |
| 15 A theorem of Jordan and Dickson | 23 |
| 16 Mathieu's group M_{22} | 25 |
| 17 The Mathieu groups, Steiner systems and coding theory | 32 |
| 18 Extension theory | 35 |
| 19 Schur's theorem | 37 |
| 20 The Higman–Sims group | 39 |
| 2 Introduction to combinatorial group theory | 45 |
| 1 Graphs and Cayley's graphs | 45 |
| 2 Automorphisms of trees | 50 |
| 3 Free groups | 52 |
| 4 The fundamental group of a graph | 56 |
| 5 Presentation of groups by generators and relations | 58 |
| 6 Tietze transformations | 60 |
| 7 A presentation of the group S_n | 63 |
| 8 Trees and free groups | 64 |
| 9 The rewriting process of Reidemeister–Schreier | 69 |
| 10 Free products | 71 |
| 11 Amalgamated free products | 72 |

| | | |
|---|---|-----|
| 12 | Trees and amalgamated free products | 74 |
| 13 | Action of the group $\mathrm{SL}_2(\mathbb{Z})$ on the hyperbolic plane | 76 |
| 14 | HNN extensions | 81 |
| 15 | Trees and HNN extensions | 84 |
| 16 | Graphs of groups and their fundamental groups | 84 |
| 17 | The relationship between amalgamated products and HNN extensions | 87 |
| 18 | The structure of a group acting on a tree | 88 |
| 19 | Kurosh's theorem | 92 |
| 20 | Coverings of graphs | 93 |
| 21 | <i>S</i> -graphs and subgroups of free groups | 96 |
| 22 | Foldings | 98 |
| 23 | The intersection of two subgroups of a free group | 101 |
| 24 | Complexes | 104 |
| 25 | Coverings of complexes | 106 |
| 26 | Surfaces | 109 |
| 27 | The theorem of Seifert and van Kampen | 115 |
| 28 | Grushko's Theorem | 115 |
| 29 | Hopfian groups and residually finite groups | 117 |
| 3 | Automorphisms of free groups and train tracks | 121 |
| 1 | Nielsen's method and generators of $\mathrm{Aut}(F_n)$ | 123 |
| 2 | Maps of graphs. Tightening, collapsing and expanding | 126 |
| 3 | Homotopy equivalences | 128 |
| 4 | Topological representatives | 129 |
| 5 | The transition matrix. Irreducible maps and automorphisms | 130 |
| 6 | Train tracks | 132 |
| 7 | Transformations of maps | 132 |
| 8 | The metric induced on a graph by an irreducible map | 137 |
| 9 | Proof of the main theorem | 138 |
| 10 | Examples of the construction of train tracks | 141 |
| 11 | Two applications of train tracks | 151 |
| Appendix. The Perron–Frobenius Theorem | | 153 |
| Solutions to selected exercises | | 157 |
| Bibliography | | 169 |
| Index | | 173 |