Preface

Differential geometry studies geometrical objects using analytical methods. Like modern analysis itself, differential geometry originates in classical mechanics. For instance, geodesics and minimal surfaces are defined via variational principles and the curvature of a curve is easily interpreted as the acceleration with respect to the path length parameter.

Modern differential geometry in its turn strongly contributed to modern physics when, for instance, at the beginning of the 20th century it was discovered by Einstein that a gravitational field is just a pseudo-Riemannian metric on space time. The basic equations of gravity theory were written in terms of the curvature of a metric, which is a geometric quantity. More recently the modern theory of elementary particles was based on gauge fields, which mathematically are connections on fiber bundles.

In this book we attempt to give an introduction to the basics of differential geometry, keeping in mind the natural origin of many geometrical quantities, as well as the applications of differential geometry and its methods to other sciences.

The book is divided into three parts. The first part covers the basics of curves and surfaces, while the second part is designed as an introduction to smooth manifolds and Riemannian geometry. In particular, in Chapter 5 we give short introductions to hyperbolic geometry and geometrical principles of special relativity theory. Here, only a basic knowledge of algebra, calculus and ordinary differential equations is required.

The third part is more advanced and assumes that the reader is familiar with the first two parts of the book. It introduces the reader to Lie groups and Lie algebras, the representation theory of groups, symplectic and Poisson geometry, and the applications of complex analysis in surface theory. We use Lie groups as important examples of smooth manifolds and we expose symplectic and Poisson geometry in their close relation with mechanics and the theory of integrable systems.

This book is a translation (with minor revisions and corrections) of Лекции по дифференциальной геометрии,[6] which is based on lecture notes[7] that arose from a one-semester course given in Novosibirsk State University, and on some advanced minicourses.

The contents and the concise style of exposition are due to the expectation that the book will be suitable for a full semester course at the upper undergraduate or beginning graduate level. During the course at Novosibirsk State University we included a complete treatment of Chapters 1, 2, and 5. Chapters 3 and 4 give a much more detailed exposition of some further material which was touched upon in the course.

[6] Lectures on Differential Geometry, 2nd ed., Moscow-Izhevsk 2006.

[7] Lectures on differential geometry. I. Curves and surfaces. Novosibirsk: NSU, 1998; II. Riemannian geometry. Novosibirsk: NSU, 1998; III. Supplement chapters. Novosibirsk: NSU, 1999.

Note that, unless specified to the contrary, repeated upper and lower indices imply summation. For example

$$a^i_k v^k := \sum_{k=1}^{n} a^i_k v^k \quad \text{where } v \in V,\ n = \dim V.$$

Although this notation rarely appears in textbooks, it is widely used in scientific publications, especially in physical applications of differential geometry. It is one of the more practical ways in which the book will be useful for students with wider interests in such applications of differential geometry.

The Bibliography is shorter than the list that appeared in the Russian edition, which contains many publications unavailable in English translation. Hence we have added supplementary reading sources in English. For further bibliography we refer to [4].

Novosibirsk, February 2008 I. A. Taimanov