Preface

Topological robotics is a new mathematical discipline studying topological problems inspired by robotics and engineering as well as problems of practical robotics requiring topological tools. It is a part of a broader newly created research area called "computational topology". The latter studies topological problems appearing in computer science and algorithmic problems in topology.

This book is based on a one-semester lecture course "Topics of Topological Robotics" which I gave at the ETH Zürich during April–June 2006. I describe here four selected mathematical stories which have interesting connections to other sciences.

Chapter 1 studies configuration spaces of mechanical linkages, a remarkable class of manifolds which appear in several fields of mathematics as well as in molecular biology and in statistical shape theory. Methods of Morse theory, enriched with new techniques based on properties of involutions, allow effective computation of their Betti numbers. We describe here a recent solution of the conjecture raised by Kevin Walker in 1985. This conjecture asserts that the relative sizes of bars of a linkage are determined, up to certain equivalence, by the cohomology algebra of the linkage configuration space.

In Chapter 1 we also discuss topology of random linkages, a probabilistic approach to topological spaces depending on a large number of random parameters.

In Chapter 2 we describe a beautiful theorem of Swiatoslaw R. Gal [**38**] which gives a general formula for Euler characteristics of configuration spaces $F(X, n)$ of of n distinct particles moving in a polyhedron X, for all n. The Euler – Gal power series is a rational function encoding all numbers $\chi(F(X, n))$ and Gal's theorem gives an explicit expression for it in terms of local topological properties of the space.

Chapter 3 deals with the knot theory of the robot arm, a variation of the traditional knot theory question motivated by robotics. The main result (which in my view is one of the remarkable jewels of modern mathematics) is an unknotting theorem for planar robot arms, proven recently by R. Connelly, E.Demaine and G. Rote [**10**].

Chapter 4 discusses the notion of topological complexity of the robot motion planning problem $\mathsf{TC}(X)$ and mentions several new results and techniques. The number $\mathsf{TC}(X)$ measures the complexity of the problem of navigation in a topological space X viewed as the configuration space of a system. In this chapter we explain how one may use stable cohomology operations to improve lower bounds on the topological complexity based on products of zero-divisors. These results were obtained jointly with Mark Grant.

I would like to thank Jean-Claude Hausmann, Thomas Kappeler and Dirk Schuetz for useful discussions of various parts of the book. My warmest thanks go also to Mark Grant who helped me in many ways to make this text readable and in particular for his advice concerning the material of Chapter 4.

Problems of topological robotics can roughly be split into two main categories: (A) studying special topological spaces, configuration spaces of important mechanical systems; (B) studying new topological invariants of general topological spaces, invariants which are motivated and inspired by applications in robotics and engineering. Class (A) includes describing the topology of varieties of linkages, configuration spaces of graphs, knot theory of the robot arm — topics which are partly covered below. The story about $\mathsf{TC}(X)$ represents a theory of class (B).

The book is intended as an appetizer and will introduce the reader to many fascinating topological problems motivated by engineering.

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