Preface

This book may be considered as a continuation of the monographs [T83], [T92], [T06]. Now we are mainly interested in spaces on domains in $\mathbb{R}^n$, related wavelet bases and wavelet frames, and extension problems. But first we deal in Chapter 1 with the usual spaces on $\mathbb{R}^n$, periodic spaces on $\mathbb{R}^n$ and on the n-torus $\mathbb{T}^n$ and their wavelet expansions under natural restrictions for the parameters involved. Spaces on arbitrary domains are the subject of Chapter 2. The heart of the exposition are the Chapters 3, 4, where we develop a theory of function spaces on so-called thick domains, including wavelet expansions and extensions to corresponding spaces on $\mathbb{R}^n$. This will be complemented in Chapter 5 by spaces on smooth manifolds and smooth domains. Finally we add in Chapter 6 a discussion about desirable properties of wavelet expansions in function spaces introducing the notation of Riesz wavelet bases and frames. This chapter deals also with some related topics, in particular with spaces on cellular domains.

Although we rely mainly on [T06] we repeat basic notation and a few classical assertions in order to make this text to some extent independently understandable and usable. More precisely, we have two types of readers in mind:

researchers in the theory of function spaces who are interested in wavelets as new effective building blocks, and

scientists who wish to use wavelet bases in *classical function spaces* in diverse applications.

Here is a ***guide*** to where one finds basic definitions and key assertions adapted to the second type of readers:

1. Classical Sobolev spaces $W^k_p(\mathbb{R}^n)$, Sobolev spaces $H^s_p(\mathbb{R}^n)$, classical Besov spaces $B^s_{pq}(\mathbb{R}^n)$ and Hölder–Zygmund spaces $\mathcal{C}^s(\mathbb{R}^n)$ on the Euclidean n-spaces $\mathbb{R}^n$: Definition 1.1, Remark 1.2, p. 2.
2. Wavelets in $\mathbb{R}^n$: Section 1.2.1, p. 13.
3. Wavelet bases for spaces on $\mathbb{R}^n$: Theorem 1.20, p. 15.
4. Spaces on the n-torus $\mathbb{T}^n$: Definition 1.27, Remark 1.28, p. 21.
5. Wavelet bases for spaces on $\mathbb{T}^n$: Theorem 1.37, p. 26.
6. Spaces on arbitrary domains Ω in $\mathbb{R}^n$: Definitions 2.1, 5.17, Remark 2.2, pp. 28, 29, 147.
7. u-wavelet systems in domains Ω in $\mathbb{R}^n$: Definitions 2.4, 6.3, pp. 32, 179.
8. u-Riesz bases and u-Riesz frames: Definition 6.5, Section 6.2.2, pp. 180, 202.

9. Wavelet bases in $L_2(\Omega)$ and $L_p(\Omega)$ in arbitrary domains Ω in $\mathbb{R}^n$: Theorems 2.33, 2.36, 2.44, pp. 49, 53, 59.

10. Classes of domains Ω in $\mathbb{R}^n$ and their relations: Definitions 3.1, 3.4, 5.40, 6.9, Proposition 3.8, pp. 70, 72, 75, 168, 182.

11. Wavelet bases in E-thick domains (covering bounded Lipschitz domains) Ω in $\mathbb{R}^n$: Theorems 3.13, 3.23, Corollary 3.25, pp. 80, 89, 91.

12. Spaces, frames and bases on manifolds: Definitions 5.1, 5.5, 5.40, Theorems 5.9, 5.37, pp. 133, 135, 136, 164, 168.

13. Frames and bases on domains: Definition 5.25, Theorems 5.27, 5.35, 5.38, 5.51, 6.7, 6.30, 6.32, 6.33, pp. 152, 153, 162, 166, 175, 181, 196, 197, 198.

Formulas are numbered within chapters. Furthermore in each chapter all definitions, theorems, propositions, corollaries and remarks are jointly and consecutively numbered. Chapter n is divided in sections $n.k$ and subsections $n.k.l$. But when quoted we refer simply to Section $n.k$ or Section $n.k.l$ instead of Section $n.k$ or Subsection $n.k.l$. If there is no danger of confusion (which is mostly the case) we write $A^s_{pq}, B^s_{pq}, F^s_{pq}, \ldots, a^s_{pq} \ldots$(spaces) instead of $A^s_{p,q}, B^s_{p,q}, F^s_{p,q}, \ldots, a^s_{p,q} \ldots$. Similarly for $a_{jm}, \lambda_{jm}, Q_{jm}$ (functions, numbers, cubes) instead of $a_{j,m}, \lambda_{j,m}, Q_{j,m}$ etc. References are ordered by names, not by labels, which roughly coincides, but may occasionally cause minor deviations. The numbers behind ► in the Bibliography mark the page(s) where the corresponding entry is quoted (with the exception of [T78]–[T06]).

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Jena, Summer 2008 Hans Triebel