

Contents

Preface	v
1 Spaces on \mathbb{R}^n and \mathbb{T}^n	1
1.1 Definitions, atoms, and local means	1
1.1.1 Definitions	1
1.1.2 Atoms	4
1.1.3 Local means	6
1.2 Spaces on \mathbb{R}^n	13
1.2.1 Wavelets in $L_2(\mathbb{R}^n)$	13
1.2.2 Wavelets in $A_{pq}^s(\mathbb{R}^n)$	14
1.2.3 Wavelets in $A_{pq}^s(\mathbb{R}^n, w)$	17
1.3 Periodic spaces on \mathbb{R}^n and \mathbb{T}^n	19
1.3.1 Definitions and basic properties	19
1.3.2 Wavelets in $A_{pq}^{s,\text{per}}(\mathbb{R}^n)$	23
1.3.3 Wavelets in $A_{pq}^s(\mathbb{T}^n)$	26
2 Spaces on arbitrary domains	28
2.1 Basic definitions	28
2.1.1 Function spaces	28
2.1.2 Wavelet systems and sequence spaces	30
2.2 Homogeneity and refined localisation spaces	33
2.2.1 Homogeneity	33
2.2.2 Pointwise multipliers	35
2.2.3 Refined localisation spaces	36
2.3 Wavelet para-bases	41
2.3.1 Some preparations	41
2.3.2 Wavelet para-bases in $F_{pq}^{s,\text{rloc}}(\Omega)$	43
2.3.3 Wavelet para-bases in $L_p(\Omega)$, $1 < p < \infty$	46
2.4 Wavelet bases	48
2.4.1 Orthonormal wavelet bases in $L_2(\Omega)$	48
2.4.2 Wavelet bases in $L_p(\Omega)$ and $F_{pq}^{s,\text{rloc}}(\Omega)$	53
2.5 Complements	55
2.5.1 Haar bases	55
2.5.2 Wavelet bases in Lorentz and Zygmund spaces	60
2.5.3 Constrained wavelet expansions for Sobolev spaces	65
3 Spaces on thick domains	69
3.1 Thick domains	69
3.1.1 Introduction	69
3.1.2 Classes of domains	69

3.1.3 Properties and examples	73
3.2 Wavelet bases in $\bar{A}_{pq}^s(\Omega)$	77
3.2.1 The spaces $\tilde{F}_{pq}^s(\Omega)$	77
3.2.2 The spaces $\bar{A}_{pq}^s(\Omega)$ I	79
3.2.3 Complemented subspaces	83
3.2.4 Porosity and smoothness zero	85
3.2.5 The spaces $\bar{A}_{pq}^s(\Omega)$ II	89
3.3 Homogeneity and refined localisation, revisited	91
3.3.1 Introduction	91
3.3.2 Homogeneity: Proof of Theorem 2.11	92
3.3.3 Wavelet bases in $F_{pq}^{s,\text{rloc}}(\Omega)$, revisited	95
3.3.4 Duality	97
4 The extension problem	101
4.1 Introduction and criterion	101
4.1.1 Introduction	101
4.1.2 A criterion	101
4.2 Main assertions	103
4.2.1 Positive smoothness	103
4.2.2 Negative smoothness	105
4.2.3 Combined smoothness	106
4.3 Complements	108
4.3.1 Interpolation	108
4.3.2 Constrained wavelet expansions in Lipschitz domains	112
4.3.3 Intrinsic characterisations	117
4.3.4 Compact embeddings	123
5 Spaces on smooth domains and manifolds	130
5.1 Wavelet frames and wavelet-friendly extensions	130
5.1.1 Introduction	130
5.1.2 Wavelet frames on manifolds	132
5.1.3 Wavelet-friendly extensions	139
5.1.4 Decompositions	147
5.1.5 Wavelet frames in domains	151
5.2 Wavelet bases: criterion and lower dimensions	158
5.2.1 Wavelet bases on manifolds	158
5.2.2 A criterion	160
5.2.3 Wavelet bases on intervals and planar domains	161
5.3 Wavelet bases: higher dimensions	163
5.3.1 Introduction	163
5.3.2 Wavelet bases on spheres and balls	164
5.3.3 Wavelet bases in cellular domains and manifolds	167
5.3.4 Wavelet bases in C^∞ domains and cellular domains	172

5.4 Wavelet frames, revisited	174
5.4.1 Wavelet frames in Lipschitz domains	174
5.4.2 Wavelet frames in (ε, δ) -domains	177
6 Complements	178
6.1 Spaces on cellular domains	178
6.1.1 Riesz bases	178
6.1.2 Basic properties	181
6.1.3 A model case: traces and extension	185
6.1.4 A model case: approximation, density, decomposition	188
6.1.5 Cubes and polyhedrons: traces and extensions	192
6.1.6 Cubes and polyhedrons: Riesz bases	196
6.1.7 Cellular domains: Riesz bases	197
6.2 Existence and non-existence of wavelet frames and bases	199
6.2.1 The role of duality, the spaces $\mathbf{B}_{pq}^s(\mathbb{R}^n)$	199
6.2.2 The non-existence of Riesz frames in exceptional spaces	202
6.2.3 Reinforced spaces	204
6.2.4 A proposal	208
6.3 Greedy bases	210
6.3.1 Definitions and basic assertions	210
6.3.2 Greedy Riesz bases	212
6.4 Dichotomy: traces versus density	215
6.4.1 Preliminaries	215
6.4.2 Traces	218
6.4.3 Dichotomy	220
6.4.4 Negative smoothness	226
6.4.5 Curiosities	226
6.4.6 Pointwise evaluation	228
6.4.7 A comment on sampling numbers	232
6.5 Polynomial reproducing formulas	237
6.5.1 Global reproducing formulas	237
6.5.2 Local reproducing formulas	239
6.5.3 A further comment on sampling numbers	240
Bibliography	243
Symbols	251
Index	255