

# Contents

<b>Prologue</b> . . . . .	1
<b>1 The Optical Structure Equations</b>	
1.1 The basic geometric setup . . . . .	29
1.2 The optical structure equations . . . . .	33
1.3 The Bianchi equations . . . . .	54
1.4 Canonical coordinate systems . . . . .	59
<b>2 The Characteristic Initial Data</b>	
2.1 The characteristic initial data . . . . .	63
2.2 Construction of the solution in an initial domain . . . . .	88
<b>3 <math>L^\infty</math> Estimates for the Connection Coefficients</b>	
3.1 Introduction . . . . .	91
3.2 $L^\infty$ estimates for $\chi'$ . . . . .	94
3.3 $L^\infty$ estimates for $\underline{\chi}'$ . . . . .	98
3.4 $L^\infty$ estimates for $\eta, \underline{\eta}$ . . . . .	102
3.5 $L^\infty$ estimates for $\omega, \underline{\omega}$ . . . . .	107
3.6 The smallness requirement on $\delta$ . . . . .	111
<b>4 <math>L^4(S)</math> Estimates for the 1st Derivatives of the Connection Coefficients</b>	
4.1 Introduction . . . . .	115
4.2 $L^4(S)$ estimates for $\nabla\chi$ . . . . .	128
4.3 $L^4(S)$ estimates for $\nabla\underline{\chi}$ . . . . .	131
4.4 $L^4(S)$ estimates for $\nabla\eta, \nabla\underline{\eta}$ . . . . .	135
4.5 $L^4(S)$ estimates for $\not{d}\omega, \not{d}\underline{\omega}$ . . . . .	141
4.6 $L^4(S)$ estimates for $D\omega, \underline{D}\omega$ . . . . .	144
<b>5 The Uniformization Theorem</b>	
5.1 Introduction. An $L^2(S)$ estimate for $K - \overline{K}$ . . . . .	147
5.2 Sobolev inequalities on $S$ . The isoperimetric constant . . . . .	153
5.3 The uniformization theorem . . . . .	162
5.4 $L^p$ elliptic theory on $S$ . . . . .	170

<b>6</b>	<b><math>L^4(S)</math> Estimates for the 2nd Derivatives of the Connection Coefficients</b>	
6.1	Introduction . . . . .	181
6.2	$L^4(S)$ estimates for $\nabla^2 \chi, K - \overline{K}$ . . . . .	182
6.3	$L^4(S)$ estimates for $\nabla^2 \underline{\chi}$ . . . . .	187
6.4	$L^4(S)$ estimates for $\nabla^2 \eta, \nabla^2 \underline{\eta}$ . . . . .	193
6.5	$L^4(S)$ estimate for $\nabla^2 \underline{\omega}$ . . . . .	206
6.6	$L^4(S)$ estimate for $\nabla^2 \omega$ . . . . .	212
<b>7</b>	<b><math>L^2</math> Estimates for the 3rd Derivatives of the Connection Coefficients</b>	
7.1	Introduction . . . . .	215
7.2	$L^2$ estimates for $\nabla^2 \eta, \nabla^2 \underline{\eta}$ . . . . .	216
7.3	$L^2$ elliptic theory for generalized Hodge systems on $S$ . . . . .	221
7.4	$L^2(S)$ estimates for $\nabla^3 \chi', \not{D}K$ . . . . .	233
7.5	$L^2$ estimates for $\nabla^3 \underline{\chi}'$ . . . . .	238
7.6	$L^2$ estimates for $\nabla^3 \eta, \nabla^3 \underline{\eta}$ . . . . .	242
7.7	$L^2$ estimate for $\nabla^3 \underline{\omega}$ . . . . .	256
7.8	$L^2$ estimates for $\nabla^2 \omega$ and $\nabla^3 \omega$ . . . . .	263
7.9	$L^2$ estimates for $\not{D}D\omega, \not{D}\underline{D}\omega, D^2\omega, \underline{D}^2\omega$ . . . . .	267
<b>8</b>	<b>The Multiplier Fields and the Commutation Fields</b>	
8.1	Introduction . . . . .	275
8.2	$L^\infty$ estimates for the deformation tensors of $L, K$ and $S$ . . . . .	277
8.3	Construction of the rotation vectorfields $O_i$ . . . . .	281
8.4	$L^\infty$ estimates for the $O_i$ and $\nabla O_i$ . . . . .	287
8.5	$L^\infty$ estimates for the deformation tensors of the $O_i$ . . . . .	292
<b>9</b>	<b>Estimates for the Derivatives of the Deformation Tensors of the Commutation Fields</b>	
9.1	Estimates for the 1st derivatives of the deformation tensors of $L, S$ . . . . .	299
9.2	Estimates for the 1st derivatives of the deformation tensors of the $O_i$ . . . . .	301
9.3	Estimates for the 2nd derivatives of the deformation tensors of $L, S$ . . . . .	313
9.4	Estimates for the 2nd derivatives of the deformation tensors of the $O_i$ . . . . .	316
<b>10</b>	<b>The Sobolev Inequalities on the <math>C_u</math> and the <math>\underline{C}_u</math></b>	
10.1	Introduction . . . . .	325
10.2	The Sobolev inequalities on the $C_u$ . . . . .	328
10.3	The Sobolev inequalities on the $\underline{C}_u$ . . . . .	336

<b>11 The <math>S</math>-tangential Derivatives and the Rotational Lie Derivatives</b>	
11.1 Introduction and preliminaries . . . . .	343
11.2 The coercivity inequalities on the standard sphere . . . . .	349
11.3 The coercivity inequalities on $S_{\underline{u},u}$ . . . . .	354
<b>12 Weyl Fields and Currents. The Existence Theorem</b>	
12.1 Weyl fields and Bianchi equations. Weyl currents . . . . .	365
12.2 Null decompositions of Weyl fields and currents . . . . .	370
12.3 The Bel-Robinson tensor. The energy-momentum density vectorfields . . . . .	377
12.4 The divergence theorem in spacetime . . . . .	378
12.5 The energies and fluxes. The quantity $\mathcal{P}_2$ . . . . .	381
12.6 The controlling quantity $\mathcal{Q}_2$ . Bootstrap assumptions and the comparison lemma . . . . .	390
12.7 Statement of the existence theorem. Outline of the continuity argument . . . . .	411
<b>13 The Multiplier Error Estimates</b>	
13.1 Preliminaries . . . . .	423
13.2 The multiplier error estimates . . . . .	428
<b>14 The 1st-Order Weyl Current Error Estimates</b>	
14.1 Introduction . . . . .	441
14.2 The error estimates arising from $J^1$ . . . . .	446
14.3 The error estimates arising from $J^2$ . . . . .	462
14.4 The error estimates arising from $J^3$ . . . . .	468
<b>15 The 2nd-Order Weyl Current Error Estimates</b>	
15.1 The 2nd-order estimates which are of the same form as the 1st-order estimates . . . . .	481
15.2 The genuine 2nd-order estimates . . . . .	493
<b>16 The Energy-Flux Estimates. Completion of the Continuity Argument</b>	
16.1 The energy-flux estimates . . . . .	517
16.2 Higher-order bounds . . . . .	521
16.3 Completion of the continuity argument . . . . .	551
16.4 Restatement of the existence theorem . . . . .	571
<b>17 Trapped Surface Formation . . . . .</b>	<b>573</b>
<b>Bibliography . . . . .</b>	<b>583</b>
<b>Index . . . . .</b>	<b>587</b>