Preface to the first edition

When J. Tits [215] introduced the abstract notion of generalized polygon in 1959, the study of finite geometries as objects of interest in their own right was hardly fashionable. But the favorable climate in which the investigation of all kinds of discrete structures thrives today was already being created by pioneers whose names are now familiar to all who work in this broad area. We mention especially B. Segre, whose work on Galois geometries (i.e. projective geometries coordinatized by the Galois fields) has exerted a tremendous positive influence for more than two decades.

The proper subject of this book – finite generalized quadrangles – originally was conceived as an adjunct to the study of finite groups, and this connection remains a healthy one today. However, the present volume may be viewed as an attempt to treat our subject as thoroughly as possible from a combinatorial and geometric point of view. Our goal has been to keep to a minimum the prerequisites needed for reading this work, while at the same time permitting the reader to develop an understanding of the many connections between generalized quadrangles and diverse other structures. In the normal course of education a graduate student would have acquired the necessary linear and abstract algebra early in his studies, and the only additional background needed is a moderate introduction to the finite affine and projective geometries. For this the recent treatise by J. W. P. Hirschfeld [80] is a convenient reference along with the text by D. R. Hughes and F. Piper [86].

In Chapter 1 nearly all the known general results of a combinatorial nature are proved using standard counting tricks, eigenvalue techniques, etc. Most of the concepts and terms that play a role throughout the book are introduced in this introductory chapter. Chapter 2 is a continuation of the general combinatorial theme as applied specifically to subquadrangles and certain other substructures such as ovoids and spreads. Then in Chapter 3 the known models are given together with a fairly detailed discussion of their properties as related to the abstract notions introduced previously. Chapter 4 is devoted to a proof of the celebrated theorem of F. Buekenhout and C. Lefèvre [29] characterizing the finite classical generalized quadrangles as those which can be embedded in projective space. The longest (and perhaps the central) chapter in the book is Chapter 5, in which are given many combinatorial characterizations of the known generalized quadrangles. Here in one unified treatment are results whose proofs (most of which are due to the younger author) were scattered in a variety of overlapping papers. The determination of all generalized quadrangles with sufficiently small parameters has been the work of several authors. This is treated in considerable detail in Chapter 6. In the next chapter there are determined all generalized quadrangles embedded in finite affine spaces, a problem first completely solved by J. A. Thas [196]. At this point the treatment becomes more algebraic.

Chapters 8 and 9 study the generalized quadrangles in terms of their collineation groups, but with a minimum of abstract group theory. These two chapters present a nearly complete elementary proof of the celebrated theorem of J. Tits determining all finite Moufang quadrangles. The authors are still hopeful that this program can be completed. Chapter 10 studies group coset geometries and gives an account of the most recently discovered family of examples, due to W. M. Kantor [89]. In the next chapter a fairly general theory of coordinates for generalized quadrangles of order s is developed. The final chapter studies generalized quadrangles as amalgamations of desarguesian planes.

We make no claim to completeness. Important work of M. Walker [228], for example, is completely ignored. But it is our hope that the present volume presents a unified, accessible treatment of a major part of the work done on finite generalized quadrangles, sufficient to obtain a firm grasp on the subject, to find inspiration to add to the current body of knowledge, and to appreciate the many interconnections with other branches of finite geometry.

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