

Contents

Introduction	xi
I Generalities on HQFTs	1
I.1 Basic definitions	1
I.2 Cohomological HQFTs and transfer	8
I.3 Aspherical targets	11
I.4 Hermitian and unitary HQFTs	16
I.5 Proof of Lemmas 1.3.1–1.3.3	17
II Group-algebras	23
II.1 G -algebras	23
II.2 Inner products and Frobenius G -algebras	24
II.3 Crossed Frobenius G -algebras	25
II.4 Transfer	28
II.5 Semisimple crossed G -algebras	31
II.6 Semisimple crossed Frobenius G -algebras	36
II.7 Hermitian G -algebras	38
III Two-dimensional HQFTs	40
III.1 The underlying G -algebra	40
III.2 Computation for cohomological HQFTs	49
III.3 Equivalence of categories	52
III.4 Proof of Theorem 3.1	54
III.5 Hermitian two-dimensional HQFTs	63
IV Biangular algebras and lattice HQFTs	66
IV.1 Frobenius G -algebras re-examined	66
IV.2 Biangular G -algebras	70
IV.3 Lattice HQFTs	74
IV.4 Skeletons of surfaces	81
IV.5 Hermitian biangular G -algebras	85
V Enumeration problems in dimension two	88
V.1 Enumeration problem for homomorphisms	88
V.2 Linear representations of Γ and cohomology	92
V.3 Projective representations of Γ	96
V.4 Properties of κ_ρ and ζ_ρ	101
V.5 Equivalence of two approaches	107
V.6 A generalization and a proof of Theorem 1.2.1	110
V.7 A homological obstruction to lifting	115

V.8	Applications of Theorem 1.2.1	121
V.9	Further applications of Theorem 1.2.1	126
VI	Crossed G-categories and invariants of links	130
VI.1	G -categories	130
VI.2	Crossed, braided, and ribbon G -categories	133
VI.3	Colored G -tangles and their invariants	140
VI.4	Colored G -graphs and their invariants	149
VI.5	Trace, dimension, and algebra of colors	154
VII	Modular G-categories and HQFTs	158
VII.1	Modular crossed G -categories	158
VII.2	Invariants of 3-dimensional G -manifolds	163
VII.3	Homotopy modular functor	170
VII.4	Two-dimensional HQFT	176
VII.5	Three-dimensional HQFT	179
VIII	Miscellaneous algebra	186
VIII.1	Hopf G -coalgebras	186
VIII.2	Canonical extensions	193
VIII.3	Transfer of categories	196
VIII.4	Quasi-abelian cohomology of groups	199
VIII.5	Remarks on group-algebras	200
Appendix 1.	Relative HQFTs	205
Appendix 2.	State sum invariants of 3-dimensional G-manifolds	213
Appendix 3.	Recent work on HQFTs	217
Appendix 4.	Open problems	219
Appendix 5.	On the structure of braided crossed G-categories	221
	(by Michael Müger)	
5.1	Braided crossed G -categories	221
5.2	The G -fixed category of a braided crossed G -category	223
5.3	From braided categories containing $\text{Rep } G$ to braided G -crossed categories	225
5.4	Classification and coherence for braided crossed G -categories	229
5.5	Braided crossed G -categories as crossed products	230
5.6	Remarks on applications in conformal field theory	233

Appendix 6. Algebraic properties of Hopf G-coalgebras	
(by Alexis Virelizier)	236
6.1 Hopf G -coalgebras	236
6.2 Quasitriangular Hopf G -coalgebras	241
6.3 The twisted double construction	245
Appendix 7. Invariants of 3-dimensional G-manifolds from Hopf coalgebras	
(by Alexis Virelizier)	253
7.1 Kuperberg-type invariants	253
7.2 Hennings–Kauffman–Radford-type invariants	258
Bibliography	263
Index	273