Preface

This book is about time-homogeneous Markov chains that evolve with discrete time steps on a countable state space. This theory was born more than 100 years ago, and its beauty stems from the simplicity of the basic concept of these random processes: "given the present, the future does not depend on the past". While of course a theory that builds upon this axiom cannot explain all the weird problems of life in our complicated world, it is coupled with an ample range of applications as well as the development of a widely ramified and fascinating mathematical theory. Markov chains provide one of the most basic models of stochastic processes that can be understood at a very elementary level, while at the same time there is an amazing amount of ongoing, new and deep research work on that subject.

The present textbook is based on my Italian lecture notes *Catene di Markov e teoria del potenziale nel discreto* from 1996 [W1]. I thank Unione Matematica Italiana for authorizing me to publish such a translation. However, this is not just a one-to-one translation. My view on the subject has widened, part of the old material has been rearranged or completely modified, and a considerable amount of material has been added. Only Chapters 1, 2, 6, 7 and 8 and a smaller portion of Chapter 3 follow closely the original, so that the material has almost doubled.

As one will see from summary (page ix) and table of contents, this is not about applied mathematics but rather tries to develop the "pure" mathematical theory, starting at a very introductory level and then displaying several of the many fascinating features of that theory.

Prerequisites are, besides the standard first year linear algebra and calculus (including power series), an understanding of and – most important – interest in probability theory, possibly including measure theory, even though a good part of the material can be digested even if measure theory is avoided. A small amount of complex function theory, in connection with the study of generating functions, is needed a few times, but only at a very light level: it is useful to know what a singularity is and that for a power series with non-negative coefficients the radius of convergence is a singularity. At some points, some elementary combinatorics is involved. For example, it will be good to know how one solves a linear recursion with constant coefficients. Besides this, very basic Hilbert space theory is needed in §C of Chapter 4, and basic topology is needed when dealing with the Martin boundary in Chapter 7. Here it is, in principle, enough to understand the topology of metric spaces.

One cannot claim that every chapter is on the same level. Some, specifically at the beginning, are more elementary, but the road is mostly uphill. I myself have used different parts of the material that is included here in courses of different levels.

The writing of the Italian lecture notes, seen *a posteriori,* was sort of a "warm up" before my monograph *Random walks on infinite graphs and groups* [W2]. Markov chain basics are treated in a rather condensed way there, and the understanding of a good part of what is expanded here in detail is what I would hope a reader could bring along for digesting that monograph.

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