

# Contents

Preface	v
<b>Introduction</b>	ix
Summary	ix
Raison d'être	xiii
<b>1 Preliminaries and basic facts</b>	1
A Preliminaries, examples	1
B Axiomatic definition of a Markov chain	5
C Transition probabilities in $n$ steps	12
D Generating functions of transition probabilities	17
<b>2 Irreducible classes</b>	28
A Irreducible and essential classes	28
B The period of an irreducible class	35
C The spectral radius of an irreducible class	39
<b>3 Recurrence and transience, convergence, and the ergodic theorem</b>	43
A Recurrent classes	43
B Return times, positive recurrence, and stationary probability measures	47
C The convergence theorem for finite Markov chains	52
D The Perron–Frobenius theorem	57
E The convergence theorem for positive recurrent Markov chains	63
F The ergodic theorem for positive recurrent Markov chains	68
G $\rho$ -recurrence	74
<b>4 Reversible Markov chains</b>	78
A The network model	78
B Speed of convergence of finite reversible Markov chains	83
C The Poincaré inequality	93
D Recurrence of infinite networks	102
E Random walks on integer lattices	109
<b>5 Models of population evolution</b>	116
A Birth-and-death Markov chains	116
B The Galton–Watson process	131
C Branching Markov chains	140

<b>6</b>	<b>Elements of the potential theory of transient Markov chains</b>	<b>153</b>
A	Motivation. The finite case . . . . .	153
B	Harmonic and superharmonic functions. Invariant and excessive measures . . . . .	158
C	Induced Markov chains . . . . .	164
D	Potentials, Riesz decomposition, approximation . . . . .	169
E	“Balayage” and domination principle . . . . .	173
<b>7</b>	<b>The Martin boundary of transient Markov chains</b>	<b>179</b>
A	Minimal harmonic functions . . . . .	179
B	The Martin compactification . . . . .	184
C	Supermartingales, superharmonic functions, and excessive measures . . . . .	191
D	The Poisson–Martin integral representation theorem . . . . .	200
E	Poisson boundary. Alternative approach to the integral representation . . . . .	209
<b>8</b>	<b>Minimal harmonic functions on Euclidean lattices</b>	<b>219</b>
<b>9</b>	<b>Nearest neighbour random walks on trees</b>	<b>226</b>
A	Basic facts and computations . . . . .	226
B	The geometric boundary of an infinite tree . . . . .	232
C	Convergence to ends and identification of the Martin boundary . . . . .	237
D	The integral representation of all harmonic functions . . . . .	246
E	Limits of harmonic functions at the boundary . . . . .	251
F	The boundary process, and the deviation from the limit geodesic . . . . .	263
G	Some recurrence/transience criteria . . . . .	267
H	Rate of escape and spectral radius . . . . .	279
	<b>Solutions of all exercises</b>	<b>297</b>
	<b>Bibliography</b>	<b>339</b>
A	Textbooks and other general references . . . . .	339
B	Research-specific references . . . . .	341
	List of symbols and notation	345
	Index	349