

Contents

Preface	v
1 Function spaces	1
1.1 Isotropic spaces	1
1.1.1 Definitions	1
1.1.2 Atoms	4
1.1.3 Local means	6
1.1.4 Wavelets	8
1.1.5 Duality and interpolation	11
1.1.6 Spaces on domains	12
1.1.7 Limiting embeddings	16
1.1.8 Spaces of measurable functions	19
1.2 Spaces with dominating mixed smoothness	22
1.2.1 Definitions	22
1.2.2 Atoms	26
1.2.3 Local means	28
1.2.4 Wavelets	30
1.2.5 Higher dimensions	33
1.2.6 Spaces on domains: definitions, problems	35
1.2.7 Spaces on domains: representations, wavelets	36
1.2.8 Spaces on domains: extensions, intrinsic characterisations	44
1.3 Logarithmic spaces	50
1.3.1 Introduction	50
1.3.2 Logarithmic spaces on \mathbb{R}	52
1.3.3 Isotropic logarithmic spaces: comments, problems, proposals	54
1.3.4 Logarithmic spaces with dominating mixed smoothness	60
2 Haar bases	63
2.1 Classical theory and historical comments	63
2.2 Haar bases on \mathbb{R} and on intervals	71
2.2.1 Introduction and plan of the chapter	71
2.2.2 Inequalities	72
2.2.3 Haar bases on \mathbb{R}	80
2.2.4 Haar bases on intervals	84
2.2.5 Littlewood–Paley theorem	86
2.3 Haar wavelet bases on \mathbb{R}^n and on cubes	88
2.3.1 Inequalities	88
2.3.2 Haar wavelet bases on \mathbb{R}^n	92
2.3.3 Haar wavelet bases on cubes	95
2.4 Haar tensor bases on \mathbb{R}^n and on cubes	98

2.4.1	Introduction	98
2.4.2	Haar tensor bases on \mathbb{R}^2 , I	99
2.4.3	Haar tensor bases on \mathbb{R}^2 , II	101
2.4.4	Haar tensor bases on cubes	106
2.4.5	Higher dimensions	109
2.5	Spline bases	112
2.5.1	Preliminaries and basic assertions	112
2.5.2	Spline bases on \mathbb{R}	115
2.5.3	Spline wavelet bases on \mathbb{R}^n	119
2.5.4	Spline tensor bases on \mathbb{R}^2	121
3	Faber bases	124
3.1	Faber bases on intervals	124
3.1.1	Introduction and preliminaries	124
3.1.2	Faber bases	126
3.1.3	Complements	130
3.2	Faber bases on cubes	133
3.2.1	Preliminaries and definitions	133
3.2.2	Faber bases in $C(\mathbb{Q}^2)$	135
3.2.3	Faber bases in $S_p^1 W(\mathbb{Q}^2)$ and $S_{pp}^r B(\mathbb{Q}^2)$	138
3.2.4	The spaces $S_{pq}^r B(\mathbb{Q}^2)$ and $S_p^r H(\mathbb{Q}^2)$	144
3.2.5	Higher dimensions	152
3.3	The spaces $S_{pq}^r \mathfrak{B}(\mathbb{Q}^2)$	154
3.3.1	Introduction and definition	154
3.3.2	Properties	157
3.4	Bases in logarithmic spaces	159
3.4.1	Preliminaries	159
3.4.2	The spaces $B_{pq}^{s,b}(\mathbb{R})$ and $B_{pq}^{s,b}(I)$	160
3.4.3	The spaces $S_{pq}^{r,b} B(\mathbb{R}^2)$ and $S_{pq}^{r,b} B(\mathbb{Q}^2)$	163
3.5	Faber splines: an outlook	165
3.5.1	Preparations	165
3.5.2	Faber splines	167
3.5.3	Comments, problems, proposals	172
4	Sampling	174
4.1	Definitions, sampling in isotropic spaces	174
4.1.1	Definitions	174
4.1.2	Sampling in isotropic spaces	176
4.1.3	Information uncertainty	181
4.2	Sampling on intervals	183
4.2.1	Sampling in $A_{pq}^s(I)$	183
4.2.2	Sampling in $A_{pq}^s(I)$	189
4.3	Sampling in spaces with dominating mixed smoothness	192

4.3.1	Introduction, preliminaries	192
4.3.2	Main assertions	195
4.3.3	Complements	203
4.4	Sampling in logarithmic spaces with dominating mixed smoothness	208
4.4.1	Introduction and motivation	208
4.4.2	Basic assertions	210
4.4.3	The spaces $S_{pq}^{r,(b)} B(\mathbb{Q}^2)$, main assertions	212
4.4.4	Higher dimensions: comments, problems, proposals	215
5	Numerical integration	221
5.1	Preliminaries, integration in domains	221
5.1.1	Introduction, definitions	221
5.1.2	Integration in Lipschitz domains	223
5.1.3	A comment on integration in E -thick domains	224
5.2	Integration in intervals	225
5.2.1	Main assertions	225
5.2.2	Comments and inequalities	228
5.3	Multivariate integration	231
5.3.1	Integration in squares	231
5.3.2	Integration in cubes	236
5.3.3	Integration based on logarithmic spaces	240
5.4	Integration in planar domains	243
5.4.1	Introduction, definitions	243
5.4.2	Spaces with dominating mixed smoothness, integration	245
6	Discrepancy	248
6.1	Introduction, definitions	248
6.1.1	Definitions	248
6.1.2	The one-dimensional case	251
6.2	Relationships between integral and discrepancy numbers	252
6.2.1	Prerequisites	252
6.2.2	Hlawka–Zaremba identity	255
6.2.3	Equivalences	257
6.3	Discrepancy numbers	258
6.3.1	Main assertions	258
6.3.2	The one-dimensional case, revisited	262
6.3.3	Comments, problems, proposals	269
	Bibliography	275
	List of Figures	289
	Symbols	291
	Index	295