

## Preface

This is an almost verbatim copy of the lecture notes from a two-quarter course I gave on *Moonshine of Finite Groups* at the Ohio State University in 1983–84.

My original motivation was to understand the moonshine phenomena of the Monster simple group  $M$ . A quarter of a century later since, now around the year 2010, it does not seem that we understand them very well, although the main problem that existed then is now a theorem. Namely, the Conway–Norton conjecture has been solved by Richard Borcherds. The idea of his proof has created a new area of mathematics, now called the theory of vertex algebras. This algebraic structure has since become a flourishing area of mathematics for many researchers around the world.

On the other hand, the interest generated by researchers in the original mystery of moonshine of the Monster simple group seems to have faded away somewhat, due perhaps to the difficulty of solving it. This is one reason why I am now making the lecture notes available for a wider body of readers. Over the last 25 years since the typed version first appeared, requests for copies of the original lecture notes became fewer but did not die.

The original version of these lecture notes included the classification of  $\Gamma_0(N)$  such that the corresponding Riemann surface  $\Gamma_0(N)\backslash H^*$  ( $H$  the upper half plane) has genus 0. This was a home-work project of one of my former students, Kamal Narang, in the class of 1983–84. It was a well-known result and therefore it has been excluded from this published version.

I considered the option of including some of the later results in order to make this publication more up-to-date and thus more valuable. However, I finally decided to keep the notes essentially the same as in the original version. As one can see in the literature, for example Gannon’s book (2006) *Moonshine beyond the Monster* and its bibliography (nearly 600 entries) [10], the horizon encircling ‘moonshine’ is vast, long and deep. It may even appear that we have yet to see a clear direction in which the truth of the moonshine will be found. But I have concluded that this kind of lecture notes should neither risk overwhelming readers with too much expansion, nor misdirect them with too little. It should be noted that many exercises are given in the text, but those are mostly theorems, propositions, or lemmas without proof.

Although I did not update the actual mathematical contents of these notes, I did make the Bibliography somewhat longer. My purpose there was to list names of the researchers in this field, but not to pick up their papers in any exhaustive way, far from it as a matter of fact. I apologize if names of important contributors to the field are not mentioned in the Bibliography.

I would like to express my sincere gratitude to C. H. Lam who scanned the original typed manuscript and painstakingly converted it to a  $\text{\TeX}$  manuscript, and also to

N. Chigira who read the manuscript and found many typographical errors. Chigira made also an improvement of one of the theorems in these notes. I would also like to thank the secretaries who typed these notes from my hand written manuscript back in the early 1980s, but their names have now been all but lost from my memory.

August 2010

Koichiro Harada