Preface

The aim of this book is to give a proof of Thurston's Geometrisation Conjecture, solved by G. Perelman in 2003. Perelman's work completes a program initiated by R. Hamilton, using a geometric evolution equation called *Ricci flow*. Perelman presented his ideas in three very concise manuscripts [Per02], [Per03a], [Per03b]. Important work has since been done to fill in the details. The first set of notes on Perelman's papers was posted on the web in June 2003 by B. Kleiner and J. Lott. These notes have progressively grown to the point where they cover the two papers [Per02], [Per03b]. The final version has been published as [KL08]. A proof of the Poincaré Conjecture, following G. Perelman, is given in the book [MT07] by J. Morgan and G. Tian. Another text covering the Geometrisation Conjecture following Perelman's ideas is the article [CZ06a] by H.-D. Cao and X.-P. Zhu. Alternative approaches to some of Perelman's arguments were given by T. Colding and W. Minicozzi [CM07], T. Shioya and T. Yamaguchi [SY05], the authors of the present book [BBB$^+$07], [BBB$^+$10], J. Morgan and G. Tian [MT08], J. Cao and J. Ge [CG09], B. Kleiner and J. Lott [KL10].

One goal of this book is to present a proof more attractive to topologists. For this purpose, we have endeavoured to reduce its analytical aspects to blackboxes and refer to some well-known and well-written sources (e.g. [MT07],[CK04]). At various points, we have favoured topological and geometric arguments over analytic ones.

The bulk of Perelman's proof is the construction and study of a kind of generalised solution of the Ricci flow equation called *Ricci flow with surgery*. In our treatment, this part has been simplified by replacing Ricci flow with surgery by a variant, which we call *Ricci flow with bubbling-off*. This is a sort of discontinuous, piecewise smooth Ricci flow on a fixed manifold. For the last part of the proof, we provide a completely different argument, based on the preprint [BBB$^+$07], which relies on topological arguments and Thurston's hyperbolisation theorem for Haken manifolds. We use a technique borrowed from the proof of the orbifold theorem ([BLP05]).

We have tried to make the various parts of the proof as independent as possible, in order to clarify its overall structure. The book has four parts. The first two are devoted to the construction of Ricci flow with bubbling-off. These two parts, combined with [CM07] or [Per03a], are sufficient to prove the Poincaré Conjecture, as we explain in Section 1.2.3. Part III is concerned with the long-time behaviour of Ricci flow with bubbling-off. Part IV completes the proof of the Geometrisation Conjecture and can be read independently of the rest. The material found here is of topological and geometric nature.

The idea of writing this book originated during the party that followed the conference held in honour of Larry Siebenmann in December 2005. It came after several workshops held in Barcelona, München and Grenoble devoted to the reading of Perelman's papers and the Kleiner–Lott notes. A first version of this book was handed out as lecture notes during the trimestre on the Ricci curvature held in I.H.P. (Paris) in May 2008.

The results of this monograph have been announced in the survey article [Mai08]. We warn the reader that there are a few discrepancies between that article and the present book, due to changes in terminology and minor adjustments in statements of theorems. For an introduction to the Poincaré Conjecture that follows the approach of this book, see [Mai09].

Many expository texts about Perelman's work have been written. We recommend, among others, [And04], [Bes06b], [Mor05], [Lot07], [Mor07], [Tao06], [Yau06], [Bes05], [Bes06a], [BBB06], [Bes07].

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