



## On the present edition

The present volume contains my translation of the *Pangeometry*, followed by some additional material.

The translation itself is accompanied by footnotes. These footnotes mostly concern the first part of the *Pangeometry*, in which Lobachevsky briefly reviewed his previous works. The notes are intended to help the reader who does not know the bases of Lobachevsky geometry to find his way more easily in the subject.

After this translation, we have reproduced the original French and Russian texts of the *Pangeometry*. This will allow the readers of the French or Russian languages to access Lobachevsky's original writings directly.

After the translation and the original texts, I have included a translation of a French biography of Lobachevsky, which was originally written as a preface to the 1886 edition of his *Collected geometric works*, published in Kazan. I wrote an introduction to this 1886 biography, and I accompanied the translation by footnotes. The introduction and the footnotes are intended to update the biography.

After this biography, I have included a commentary on the *Pangeometry*. It is intended to help the interested reader to get a quick overview on the whole memoir, and to put this memoir in a larger perspective. The commentary concerns some mathematical points, as well as historical material and references.

My translation of the *Pangeometry* is not completely literal. The reason is that, as I have already said, I think the *Pangeometry* will be read not only for its historical value, but also by geometers and students in geometry who want to learn more about the subject. Therefore I made a few minor formal changes that will make its reading easier. Apart from the formal changes that are listed below, I tried to be most faithful to the original text.

Here is the list of changes that I made:

- I highlighted certain definitions (angle of parallelism, side of parallelism, limit circle, etc.) by putting the corresponding words in italics.
- I corrected a few misprints, and I included some small corrections that were done in the German 1902 edition by Liebmann.
- I added figures to the text. The original text does not contain figures. The German edition by Liebmann contains figures, and some of my figures are the same as Liebmann's.
- Lobachevsky used the word "line" as a generic word denoting a segment, the length of a segment, and sometimes a distance. It was still common, in nineteenth-century mathematical writing, to identify a segment with its length.<sup>4</sup> To render the text clearer to the modern reader, I used the terms "segment", "length" and "distance" when appropriate.
- Likewise, Lobachevsky used the word "surface" to denote "area", and he sometimes talked about the "value" of a region, to designate its area. I used the word "area" when appropriate.

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<sup>4</sup>This is also the case in Euclid's *Elements*.

- Lobachevsky's sentences in French are often very long. For clarity of exposition, and since having long sentences is unusual in English, I divided certain long sentences into several short ones. In particular, I divided long sentences containing several definitions into shorter sentences, each containing an individual definition. For the same reasons, I divided some paragraphs into shorter ones, when the same paragraph deals with several ideas.

- I sometimes replaced the future tense by the present, when the future was not necessary. For instance, instead of "the two quantities will be equal", I wrote "the two quantities are equal".

- I added an index (see page 78).

Except for these non-significant details, the terminology and the notation used in this memoir, which is more than 150 years old, are comparable with those of an extremely well-written modern mathematical text. This is probably not so surprising, due to the fact that Lobachevsky was ahead of his time in every sense of the word.