Preface

As a result of the incorrect statement by A. Cauchy in his book *Cours d'Analyse* (1821) that any separately continuous function on $\mathbb{R} \times \mathbb{R}$ is in fact continuous, a discussion was generated as to whether certain separate regularity properties for functions in several variables are in fact regularity properties with respect to all variables simultaneously. While in the case of continuous functions this kind of implication fails to hold, the situation of holomorphic functions is much better:

Any separately holomorphic function in several complex variables is automatically holomorphic as a function of all variables.

This deep result was proved by Friedrich Hartogs in 1906. Since that time, generalizations of this result in various directions have been intensively pursued. Now, more than a hundred years after the seminal work by Hartogs, we felt that the time had come to gather some of the main streams of these developments into one single source book. Since we have also been working in that area, we reached the conclusion that it might even be our duty to complete such a project. The book may thus be understood as a kind of hundredth anniversary of separate holomorphicity.

The book is divided into two parts. The first one, which is more elementary, deals with separately holomorphic functions "without singularities", while in the second part the situation of existing singularities is discussed. For more details, the reader is asked to consult the Introduction.

Holomorphic extension problems are important questions in several complex variables (SCV). Therefore, the book should be interesting for anyone seeking further knowledge on this type of phenomena, in particular, for students (if they have attended a course on SCV). We are confident that a part of this book will serve as a basis for seminars on several complex variables and that other parts will lead to further research.

We do not claim that the text is easy (especially in Part II). However, to help the reader as much as possible, much of the necessary prerequisite knowledge has been collected in Chapter 3 (most of it without proofs but with hints on where to find them). Moreover, most later sections begin with $\boxed{>}$ § … showing which part of the previous sections may be helpful in understanding the current one. As an orientation for the reader, each chapter starts with a short summary of the topics one may find in it.

There are a few theorems for which no proofs are given; they are denoted by Theorem*. Moreover, there are many points in the proofs that we have marked EXERCISE. By this we mean that the reader is encouraged to write out the argument in more detail than we have done. At the end of the book a list of general symbols with short explanations is found; in addition, each chapter has its own list of symbols introduced in it. Finally, from time to time we pose open problems (marked by $\boxed{?}$ … $\boxed{?}$) that to the best of our knowledge have not yet been solved. We encourage the reader to try to solve them.

We want to point out that we, the authors, are responsible for all mistakes that may be still in the text. It would be much appreciated by us if the readers could inform us about any mistakes they have found, using our email addresses:

Marek.Jarnicki@im.uj.edu.pl, Peter.Pflug@uni-oldenburg.de.

Finally, it is our great pleasure to thank the following institutions for their support during the writing of this book:
Jagiellonian University in Kraków,
Carl von Ossietzky Universität Oldenburg,
Polish Ministry of Science and Higher Education – grant N N201 361436,
Deutsche Forschungsgemeinschaft – grant 436POL113/103/0-2.
Without this help it would have been impossible to finish this project.

We are deeply indebted to Dr. M. Karbe – Publishing Director of the EMS Publishing House – for his constant help and to Dr. I. Zimmermann for taking care of the text.

Kraków–Oldenburg, June 2011

Marek Jarnicki
Peter Pflug