Preface

This book is an exposition of some basic algebraic topology at the border between homology and homotopy. The emphasis is explained as follows. A Leverhulme Emeritus Fellowship was awarded to the first author for 2002–2004 to support making available, in one volume and in a consistent style, the work on crossed complexes and related higher homotopy groupoids carried out from 1974 to 2005 mainly by the first two authors. This work resulted in 12 joint papers as well as a number of other collaborations.

The project arose from the question formulated in about 1965 as to whether or not groupoids could be useful in higher homotopy theory. Could one develop theories and applications of higher groupoids in a spirit similar to that of combinatorial group theory, enabling both understanding and calculation, and thus continuing J. H. C. Whitehead's project of 'Combinatorial homotopy'? This aim also explains the term 'Higher dimensional group theory' in [Bro82].

This account also elucidates fully, as did [BH81a], a paragraph near the end of the Introduction to [Bro67] which mentioned an n-dimensional version of the Seifert–van Kampen Theorem, but really referred speculatively to an intuition of a proof, still then in search of a theorem. The necessary machinery of strict cubical higher homotopy groupoids was set up over the years to express that intuition. Surprisingly, it worked out as intended, though the task involved many new ideas and inputs from many people. However the key notion of *higher homotopy groupoid* has not attracted much attention in algebraic topology. So a full exposition is needed.

We also try to include references to work directly relevant to the main themes, and hope this book will also be useful as a reference on related work. It is not intended to be a survey of all work on, for example, crossed modules. Also we cannot claim that the historical references are full and definitive, but we hope they will give a useful entrée to the literature.

The organisation and some new details have been worked out by Brown and Sivera who carry full responsibility for the final result and in particular for errors and obscurities. However the main thrust of this exposition comes from the joint papers of Brown and Higgins; the contribution of Philip Higgins to this research by imagination, powers of organisation of material, algebraic insight and expository skills are seen throughout this book, and so he is rightly a joint author.

Obtaining these results depended on other fortunate collaborations, particularly initial work on double groupoids and crossed modules with Chris Spencer in 1971–73 under SERC support, see [BS76a], [BS76b]. Collaboration at Bangor over the years with Tim Porter and Chris Wensley has been especially important. Other collaborators on joint papers relevant to the 'groupoid project' were: Lew Hardy, Jean-Louis Loday, Sid Morris, Phil Heath, Peter Booth, Johannes Huebschmann, Graham Ellis, Heiner Kamps, Nick Gilbert, Tim Porter, David Johnson, Edmund Robertson, Hans Baues,

Razak Salleh, Kirill Mackenzie, Marek Golasinski, Mohammed Aof, Rafael Sivera, Osman Mucuk, George Janelidze, Ilhan Icen, James Glazebrook; research students at Bangor (with date of completion, supervised by Brown unless marked P for Porter or W for Wensley): Lew Hardy (1974), Tony Seda (1974), A. Razak Salleh (1975), Keith Dakin (1976), Nick Ashley (1978), David Jones (1984), Graham Ellis (1984), Fahmi Korkes (1985, P), Ghafar H. Mosa (1987), Mohammed Aof (1988), Fahd Al-Agl (1988), Osman Mucuk (1993), Andy Tonks (1993), Ilhan Icen (1996), Phil Ehlers (1994, P), J. Shrimpton (1990, with W), Zaki Arvasi (1995, P), Murat Alp (1997, W), Ali Mutlu (1998, W), Anne Heyworth (1998 with W), Emma Moore (2001, with W).

It is a pleasure to acknowledge also:

(i) The influence of the work of Henry Whitehead, who was Brown's supervisor until 1960, when Henry died suddenly in Princeton at the age of 55. It was then that Michael Barratt guided Brown's doctoral work towards the homotopy type of function spaces, a study whose methods are in the background of much of this book; Michael's splendid example of how to go about mathematical research is gratefully acknowledged. In writing the first edition of the book which has now become 'Topology and Groupoids', [Bro06], Brown turned to Whitehead's work on 'Combinatorial Homotopy'; that term shows the influence on Whitehead of the analogy between ideas in homotopy theory and in combinatorial group theory.

(ii) The further contributions of research students at Bangor, and of other colleagues, who all contributed key ideas to the whole programme.

(iii) The stimulus of a correspondence with Alexander Grothendieck in the years 1982–1991. This correspondence is to be published by the Société Mathématique de France, as an Appendix to the visionary manuscript by Grothendieck, 'Pursuing Stacks', which was distributed from Bangor in 1983, see [GroPS1], [GroPS2].[1]

(iv) The support of the Leverhulme Trust through an Emeritus Fellowship for Brown in 2002–2004, and to my chosen referees for that, Professors A. Bak (Bielefeld) and J. P. May (Chicago). This Fellowship provided the support for: the revision and publication of 'Topology and Groupoids'; meetings of Brown with Sivera and with Higgins; LaTeX work, well done by Genevieve Tan and Peilang Wu; other travel and visitors; and also a moral impetus to complete this project.

(v) Support by the University of Wales and by the SERC for the collaboration with Higgins in the years 1974–1985, and the SERC/EPSRC for support of 11 of the above research students, including Keith Dakin, whose invention of T-complexes, [Dak77], made it clear that there did exist acceptable generalisations to all dimensions of double groupoids with connections.

(vi) Chris Wensley for careful reading of many parts of drafts of this book, and seeing errors and obscurities; for the joint published work involved in Chapter 5; and supplying Section 5.9 on the use of symbolic computation for computing induced crossed modules.

(vii) Sergei Soloviev for work with Brown on Section C.7 of Appendix C on monoidal closed categories.

(viii) Jean-Louis Loday for a happy collaboration starting in 1981 and resulting in

three joint papers which showed how aspects of these ideas could be taken further.

(ix) Johannes Huebschmann for helpful remarks on the history of identities among relations and the influence of Seifert and Reidemeister; Urs Schreiber for comments on the Dold–Kan Theorem; and Ulrich Diez for writing the macro which gives the link between endnotes and hyperref.

(x) Manfred Karbe for guidance of the book through to this form, and Irene Zimmermann for tremendous and perceptive help at the copy editing stage.

We have of course tried to avoid typos, mistakes, obscurities and infelicities, and failure to reference appropriate material, but are unlikely to have done so completely. We therefore thank in advance those readers who inform us of such, and intend to keep a note of these on the web.

We thank many for support and interest, and also those whose unexplained scepticism, even disbelief, has over decades been a challenge and stimulus.

One aim of this account has been to give appropriate honour to the structures which arise naturally from the geometry, and thus make it easier to find further developments of these methods. For this reason, the final Chapter 16 on 'Future directions?' suggests a number of problems and questions.

Notes

1 p. xiv In a letter dated 02/05/1983 Alexander Grothendieck wrote:

> Don't be surprised by my supposed efficiency in digging out the right kind of notions – I have just been following, rather let myself be pulled ahead, by that very strong thread (roughly: understand non commutative cohomology of topoi!) which I kept trying to sell for about ten or twenty years now, without anyone ready to "buy" it, namely to do the work. So finally I got mad and decided to work out at least an outline by myself.