Prerequisites and reading plan

The aim is for the major parts of this book to be readable by a graduate student acquainted with general topology, the fundamental group, notions of homotopy, and some basic methods of category theory. Many of these areas, including the concept of groupoid and its uses, are covered in Brown's text 'Topology and Groupoids', [Bro06]. The only theory we have to assume for the Homotopy Classification Theorem in Chapter 11 is some results on the geometric realisation of cubical sets.

Some aspects of category theory perhaps less familiar to a graduate student, or for which we wish to emphasise a viewpoint, are given in Appendices, particularly the notion of representable functor, the notion of dense subcategory, and the preservation of colimits by a left adjoint functor. This last fact is a simple but basic tool of algebraic computation for those algebraic structures which are built up in several levels, since it can often show that a colimit of such a structure can be built up level by level. We also give an account of fibrations and cofibrations of categories, which give a general background to the notions and techniques for dealing with colimits of mathematical structures with structures at various levels. Indeed it is *the use in algebraic topology of algebraic colimit arguments rather than exact sequences that is a key feature of this book*.

We make no use of classical tools such as simplicial approximation, but some knowledge of homology and homotopy of chain complexes could be useful at a few points, to help motivate some definitions.

We feel it is important for readers to understand how this theory derives from the basic intuitions and history of algebraic topology, and so we start Part I with some history. After that, historical comments are given in Notes at the end of each chapter.

This book is designed to cater for a variety of readers.

Those with some familiarity with traditional accounts of relative homotopy theory could skip through the first two chapters, and then turn to Chapter 6, and its key account of the homotopy double groupoid $\rho(X, A, C)$ of a pair of spaces (X, A) with a set C of base points. This construction avoids two problems with second relative homotopy groups, namely that a choice has to be made in their definition, and all their group compositions are on a single line. The quite natural construction of the homotopy double groupoid is the key to proving a 2-dimensional Seifert–van Kampen Theorem, and so giving the applications in Chapters 4 and 5. Part I does develop a lot of the algebra and applications of crossed modules (particularly coproducts and induced crossed modules) and the full story of these can be skipped over.

Part II gives the major applications of crossed complexes, with the proofs of key results given in Part III, using the techniques of cubical ω-groupoids.

Finally, those who want the pure logical order could read the book starting with Part III, and referring back for basic definitions where necessary; or, perhaps better, start with Chapter 6 of Part I for the pictures and intuitions, and then turn to Part III.

The book [Bro06] is used as a basic source for background material on groupoids in homotopy theory. Otherwise, in order not to interrupt the flow of the text, and to give an opportunity for wider comment, we have put most background comments and references to the literature in notes at the end of each chapter. Nonetheless we must make the usual apology that we might not have been fair to all contributors to the subject and area.

We have tried to make the index as useful as possible, by indexing multi-part terms under each part. To make the Bibliography more useful, we have used hyperref to list the pages on which an item is cited; thus the Bibliography serves to some extent as a name index.

Because of the complexity and intricacy of the structure we present, readers may find it useful to have the e-version with hyperref as well as the printed version.