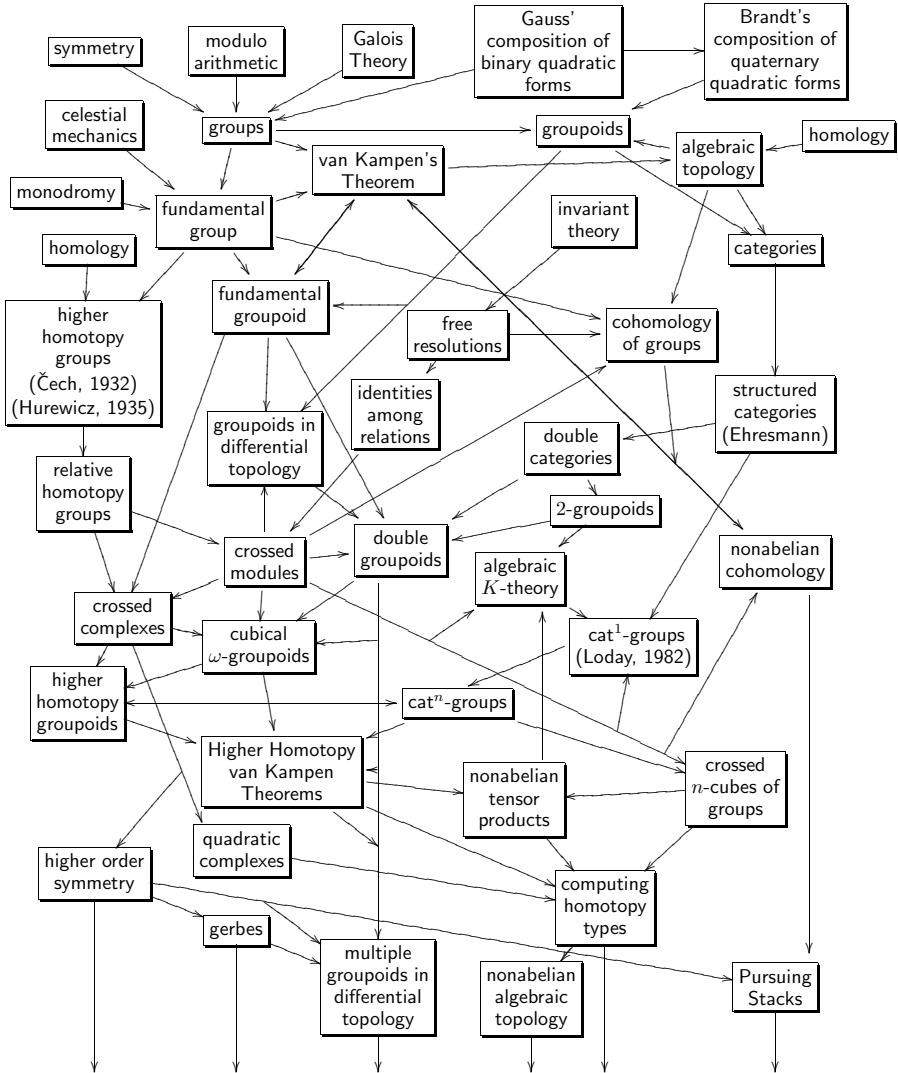


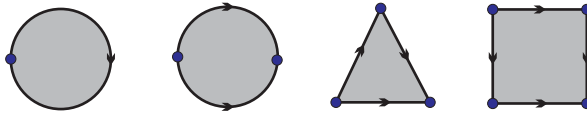
# Historical context diagram



This diagram aims to give a sketch of some influences and interactions leading to the development of nonabelian algebraic topology, and higher dimensional algebra, so that this exposition is seen as part of a continuing development. There are a number of other inputs and directions which were not easy to fit in the diagram, for example the

contribution of Seifert to the Seifert–van Kampen Theorem, and the work of C. Ehresmann on  $n$ -fold categories. He was a pioneer in this, and his definition and examples of double categories in [Ehr65] were in 1965 a starting input for this project on groupoids in higher homotopy theory.

The theory of groupoids and categories gets more complicated in higher dimensions basically because of the complexity of the basic geometric objects. Thus in dimension 2 we might take as basic objects the 2-disk, 2-globe, 2-simplex, or 2-cube as in the following pictures:



In this book we will use principally the 2-disks, which give us crossed modules, and the 2-cubes, which give us double groupoids, while in higher dimensions the disks and cubes give us crossed complexes and cubical  $\omega$ -groupoids respectively. In essence, the cubical model leads to conjectures and then theorems, partly through the ease of expressing multiple compositions, see p. xxii, and Remark 6.3.2, while the disk model leads to calculations, and clear relations to classical work.

In category theory rather than groupoid theory the disk model is not available. There is however important work, even a majority, in higher category theory which takes a globular or simplicial rather than cubical route, so there is still much work to be done to relate and evaluate all these models, for the aims of this treatise, or for current and future applications.