

Contents

Preface	v
Introduction	vii
1 $\mathbb{Z}/2\mathbb{Z}$-graded linear algebra	1
1.1 Super vector spaces and superalgebras	1
1.2 Super Lie algebras	8
1.3 Modules for superalgebras	9
1.4 The language of matrices	10
1.5 The Berezinian	12
1.6 The universal enveloping superalgebra	16
1.7 Hopf superalgebras	25
1.8 The even rules	26
1.9 References	27
2 Sheaves, functors and the geometric point of view	28
2.1 Ringed spaces of functions	28
2.2 Sheaves and ringed spaces	30
2.3 Schemes	35
2.4 Functor of points	38
2.5 Coherent sheaves	43
2.6 References	44
3 Supergroupes	45
3.1 Superspaces	45
3.2 Supermanifolds	48
3.3 Superschemes	50
3.4 The functor of points	51
3.5 References	53
4 Differentiable supermanifolds	54
4.1 Superdomains and their morphisms	54
4.2 The category of supermanifolds	59
4.3 Local and infinitesimal theory of supermanifolds	65
4.4 Vector fields and differential operators	70
4.5 Global aspects of smooth supermanifolds	74
4.6 The functor of points of supermanifolds	79
4.7 Distributions with finite support	82

4.8	Complex and real supermanifolds	86
4.9	References	89
5	The local structure of morphisms	90
5.1	The inverse function theorem	90
5.2	Immersions, submersions and the constant rank morphisms	92
5.3	Submanifolds	97
5.4	References	102
6	The Frobenius theorem	103
6.1	The local super Frobenius theorem	103
6.2	The global super Frobenius theorem	110
6.3	References	111
7	Super Lie groups	112
7.1	Super Lie groups	112
7.2	The super Lie algebra of a super Lie group	114
7.3	The Hopf superalgebra of distributions	118
7.4	Super Harish-Chandra pairs	123
7.5	Homogeneous one-parameter supergroups	137
7.6	References	140
8	Actions of super Lie groups	141
8.1	Actions of super Lie groups on supermanifolds	141
8.2	Infinitesimal actions	144
8.3	Actions of super Harish-Chandra pairs	146
8.4	The stabilizer subgroup	149
8.5	References	153
9	Homogeneous spaces	154
9.1	Transitive actions	154
9.2	Homogeneous spaces: The classical construction	157
9.3	Homogeneous superspaces for super Lie groups	158
9.4	The functor of points of a quotient supermanifold	162
9.5	The super Minkowski and super conformal spacetime	166
9.6	References	172
10	Supervarieties and superschemes	173
10.1	Basic definitions	173
10.2	The functor of points	179
10.3	A representability criterion	183
10.4	The Grassmannian superscheme	190
10.5	Projective supereometry	192
10.6	The infinitesimal theory	196

10.7 References	201
11 Algebraic supergroups	202
11.1 Supergroup functors and supergroup schemes	202
11.2 Lie superalgebras	207
11.3 $\text{Lie}(G)$ of a supergroup functor G	210
11.4 $\text{Lie}(G)$ for a supergroup scheme G	213
11.5 The Lie superalgebra of a supergroup scheme	215
11.6 Affine algebraic supergroups	221
11.7 Linear representations	223
11.8 The algebraic stabilizer theorem	227
11.9 References	230
Appendices (with the assistance of Ivan Dimitrov)	231
A Lie superalgebras	231
A.1 Classical Lie superalgebras	232
A.2 Root systems	237
A.3 Cartan matrices and Dynkin diagrams	242
A.4 Classification of finite-dimensional irreducible modules	244
A.5 Representations of basic Lie superalgebras	248
A.6 More on representations of Lie superalgebras	254
A.7 Schur's lemma	256
B Categories	259
B.1 Categories	259
B.2 Sheafification of a functor	262
B.3 Super Nakayama's lemma and projective modules	266
B.4 References	268
C Fréchet superspaces	269
C.1 Fréchet spaces	269
C.2 Fréchet superspaces	272
Bibliography	277
Index	283