

*Il faut toujours dire ce que l'on voit.  
Surtout il faut toujours, ce qui est  
plus difficile, voir ce que l'on voit.*

Charles Péguy, *Notre Jeunesse*, 1910.

## Foreword

This series of lectures on proof-theory *a priori* addresses mathematicians and computer-scientists, physicists, philosophers and linguists; and, since we are no longer in the XVI<sup>th</sup> – not to speak of the XVIII<sup>th</sup> – century, it is doomed to failure. Such a prediction is in contrast to a course focusing on subdomains which work quite well (model-theory, set-theory), not that well (temporal or modal logics), or not at all (quantum or epistemic logics) and which would therefore be grounded on a certain technical excellence or, more prosaically, on a well-understood circle of scientific welfare. This being said, plain success is not the only possible goal; mine might simply be the exposition of a disorder in this apparently well-organised universe, in which logic eventually takes its place between two beer mugs and the *Reader's Digest* and no longer disturbs anybody – like a fat cat purring on the carpet.

On the eve of the last century, the cat was rather a wolf-dog, of the strongly barking kind; the XX<sup>th</sup> century has been a century of totalitarianisms of all possible kinds, in particular the linguistic variety (styled « turn »). This extreme form of *scientism* consisted in the reduction of any mathematical question (therefore, everything being supposedly mathematisable, any question) to a problem of formal, linguistic, bureaucratic, protocols: Kafka was waiting behind the door. Dating back to 1904, the same scientism was involved in improvement of the human species in Namibia, at the hands of the II<sup>nd</sup> Reich of the blueprint of the *final solution* proper: how many gallows in this treeless country! Modern logic remains basically impregnated with the « 1900 spirit », this sort of pretension at simplifying everything, since one can solve all problems. When, after 1930, incompleteness shook this haughtiness, one hardly observed more than a complexification of the discourse: instead of explaining from the simpler, one started to explain from the « meta ». There began the time of counterfeit coinage. Since that time, logic, unable to effect its own reformation, severed its links with mathematics, physics, etc.

A typical sophism: what is the point of seeking beautiful mathematical structures for logic? Such a thing cannot exist, since, as mathematics, good or bad, can be translated into logic, the logical structure must reflect the worst, i.e., not exist or, at least, remain very bleak. For instance, when looking for a topological, continuous, interpretation of logic, one will head for the worst (e.g., Scott domains) and one will even be proud of it! Among the revealing details is the insistence of logicians on choosing counter-intuitive symbols, in order to make sure that one does not suggest

that certain properties – say distributivity – might be more important than others<sup>1</sup>: « More important, really ? How do you define importance ? ». This reminds me of my daughter Isabelle – then very young – « Why not call the door “spoon” and the spoon “door” ? », to which I answered « When one says “Make for the door”, it should not be taken as an invitation to supper ». Among the magisterial mistakes of logic, one will first mention quantum logic, whose ridiculousness can only be ascribed to a feeling of superiority of the language – and ideas, even bad, as soon as they take a written form – over the physical world. Quantum logic is indeed a sort of punishment inflicted on nature, guilty of not yielding to the prejudices of logicians... just like Xerxes had the Hellespont – which had destroyed a boat bridge – whipped.

One century ago, very scarce were those daring to oppose scientists’ certainties. After one century of slaughter, this is now much easier: even if the same baloney sempiternally comes back, like the intelligent robot, a fantasy of Artificial Intelligence and unlikely prosthesis for those who badly need it, we have won the right to make fun of scientific *Jivaros*. An instance of this is H. Simon, the guy who had his computer « rediscover » Kepler’s third law (squares and cubes), forgetting that it is not the law linking the period and the semimajor axis which is hard to find, it is the very idea of such a law, especially for an... astrologer like Kepler.

It would be fair to observe that, in spite of its heavy scientist-created liabilities, the domain of logic, although limited, is not empty. Model-theory and set-theory are doing rather well; even proof-theory has a non-negligible place and, by the way, what would I otherwise start from, since my topic will precisely be *proof-theory*?

At the beginning of the last century, Einstein’s relativity and, in a more radical way, quantum physics, called in question our « fundamental intuitions ». Logic, because of its excesses, decided to catapult itself into emptiness; the non-structure, the non-significant « Everything can be coded in everything<sup>2</sup>, and also into the sea of the idea of translating images into sound, or rather gurglings ! ». Still, in the « linguistic turn », the idea of pregnancy of the language was deeply inspired and didn’t deserve to become this « machine à décerveler<sup>3</sup> » that we just mentioned. With a closer look, the pregnancy of language contains the germ of another form of « relativisation », in fact of *derealisation* of nature. This is the viewpoint I will try to develop.

One has the right to find this project crazy and to prefer a preamble of the style « *A language is a finite alphabet with which one constructs terms, formulas, proofs – syntax; the language is in turn interpreted in a model – semantics; eventually, this is formalised in a meta-system.* ». But then one does not do logic, at least

<sup>1</sup>Witness for instance, on the eve of linear logic, the point of honour taken by those who insisted on writing « par » + and « with » ×, while « par » distributes over « times ».

<sup>2</sup>And conversely, I suppose ! The idea of mutual codings is ancient and universal : think of des Esseintes and his *orgue à liqueurs* (Huysmans, *À rebours*, 1885).

<sup>3</sup>Removal of the brain, according to Alfred Jarry.

not foundations: why not undertake to seal in the Bering strait? The domain, as it ossified during the XX<sup>th</sup> century is indeed everything but crazy: a cemetery of ideas. In other words, the only excuse in the XXI<sup>th</sup> century for indulging in « foundations », is a « grain de folie », i.e., a slight madness.

About the title: it was while revising the text (Summer 2005) that I noticed the recurrence of the expression « blind spot ». The blind spot is what one does not see and what one is not even conscious of not seeing<sup>4</sup>. The most trivial blind spot is the cheap modal logic justified by an even cheaper Kripke semantics and *vice versa*; but one finds similar blindings in the most elaborated interpretations. The good news of these lectures is that the *procedural* standpoint seems to be capable of dislodging the unsaid, the unseen. Simply, while the absence of *Hauptsatz* is enough to show that logic **S5** is nonsense, one has to work much more to imagine what could be wrong in the principles justifying – say – the function  $2^n$ .

**Acknowledgements.** This indeed constitutes the notes of a series of lectures given at the Université Franco-Italienne, from October to December 2004, at Università Roma Tre (Dipartimento di Filosofia). I am very grateful to Michele Abrusci who initiated this course. I particularly thank, for their help, Lorenzo Tortora de Falco and Marco Pedicini; and, – last but not least – the « linchpin » of the whole enterprise, Roberto Maieli, who took care of the organisation of lessons, of the video *streamings*, of the maintenance of the text on the Web.

Three months of lecturing together with the simultaneous composition of lectures notes of 500-odd pages is a very very heavy burden taking six concentrated days per week for the duration of the course. I would never have succeeded without the support of the audience, not very numerous, but fervent. Nor without the constant attention of Louise.

Two additional subsections, 15.C and 15.D have essentially been written by Olivier Laurent. The definitive version takes into account corrections suggested by Thomas Streicher (Chapter 2), Philip Scott (Chapter 8) and the long list of typos found by Akim Demaille.

This English translation owes much to the enthusiasm of Manfred Karbe, who convinced me to translate the French original in English; this is indeed much more than a translation, since the last three chapters have been completely rewritten, thus take care of the latest developments (2011). The book was carefully reread by Edwin Beschler who not only expunged the gallicisms but also clarified the text, so that it is at points superior to the French original. The typesetting is due to Irene Zimmermann; heavy work, but the result is superb!

---

<sup>4</sup>Kreisel in 1984, speaking of certain Americans: « They have no soul and they don't know that they have none ».