

Contents

Preface	vii
1 Generalized quadrangles	1
1.1 Elementary combinatorial preliminaries	1
1.2 Some group theory	9
1.3 Finite projective geometry	12
1.4 Finite classical examples and their duals	13
2 The Moufang condition	16
2.1 Moufang quadrangles	16
2.2 Generators and relations	17
2.3 Coxeter groups	18
2.4 BN-pairs of rank 2 and quadrangles	19
3 Elation quadrangles	24
3.1 Automorphisms of classical quadrangles	24
3.2 Elation generalized quadrangles	26
3.3 Maximality and completeness	27
3.4 Kantor families	27
3.5 The classical GQs as EGQs – second approach	29
4 Some features of special p-groups	31
4.1 The general Heisenberg group	31
4.2 Exact sequences and complexes	32
4.3 Group cohomology	35
4.4 Special and extra-special p -groups	37
4.5 Another approach	38
4.6 Lie algebras	39
4.7 Lie algebras from p -groups	41
5 Parameters of elation quadrangles and structure of elation groups	44
5.1 Parameters of elation quadrangles	44
5.2 Skew translation quadrangles	46
5.3 \mathcal{F} -Factors	47
5.4 Parameters of STGQs	49

6	Standard elations and flock quadrangles	50
6.1	Flock quadrangles	50
6.2	Fundamental theorem of q -clan geometry	52
6.3	A special elation	55
6.4	The nitty gritty	55
6.5	A special elation, once again	57
6.6	Standard elations in flock GQs	58
6.7	The general case	62
7	Foundations of EGQs	64
7.1	An application of Burnside's lemma	64
7.2	Implications	66
7.3	Intermezzo – SPGQs	67
7.4	The classical and dual classical examples	68
7.5	Elation groups for flock GQs and their duals	69
7.6	Dual TGQs which are also EGQs	70
7.7	GQs of order $(k - 1, k + 1)$ and their duals	76
8	Elation quadrangles with nonisomorphic elation groups	78
8.1	A nonisomorphism criterion	78
8.2	An example: $H(3, q^2)$, q even	81
8.3	Group and GQ automorphisms	81
8.4	Appendix: GQs not having property (*)	82
9	Application: Existence of translation nets	84
9.1	Translation nets	84
9.2	Construction	84
10	Elations of dual translation quadrangles	86
10.1	Main result	86
10.2	Payne's question in a more general setting	88
10.3	Recent results	88
11	Local Moufang conditions	91
11.1	Formulation	91
11.2	Proof of the first main theorem	92
11.3	Solution of Knarr's question	100
11.4	Appendix: GQs with a center of transitivity (and $s \leq t$)	100
	Bibliography	105
	Symbols	111
	Index	113