

# Contents

Preface	vii
Notation and conventions	xiii
<b>1 Metric spaces and large scale geometry</b>	1
1.1 Metric spaces . . . . .	1
1.2 Groups as metric spaces . . . . .	6
1.3 Quasi-isometries . . . . .	9
1.4 Coarse equivalences . . . . .	15
1.5 Hyperbolic spaces . . . . .	19
Exercises . . . . .	24
Notes and remarks . . . . .	25
<b>2 Asymptotic dimension and decomposition complexity</b>	26
2.1 Topological dimension . . . . .	26
2.2 Asymptotic dimension . . . . .	27
2.3 Dimension of hyperbolic groups . . . . .	30
2.4 Upper bounds for asymptotic dimension . . . . .	32
2.5 Asymptotic dimension of solvable groups . . . . .	36
2.6 Groups with infinite asymptotic dimension . . . . .	38
2.7 Decomposition complexity . . . . .	39
2.8 Invariance and permanence . . . . .	42
2.9 Groups with finite decomposition complexity . . . . .	45
Exercises . . . . .	47
Notes and remarks . . . . .	47
<b>3 Amenability</b>	49
3.1 Følner conditions . . . . .	49
3.2 The Hulanicki–Reiter condition . . . . .	55
3.3 Invariant means . . . . .	58
Exercises . . . . .	61
Notes and remarks . . . . .	61
<b>4 Property A</b>	63
4.1 Definition and basic properties . . . . .	63
4.2 The Higson–Roe condition . . . . .	66
4.3 Finite asymptotic dimension implies property A . . . . .	70

4.4	Property A and residually finite groups . . . . .	73
4.5	Locally finite examples . . . . .	78
	Exercises . . . . .	81
	Notes and remarks . . . . .	81
<b>5</b>	<b>Coarse embeddings</b>	83
5.1	Coarse embeddings . . . . .	83
5.2	Embeddability into Hilbert spaces . . . . .	84
5.3	Examples of embeddable spaces without property A . . . . .	89
5.4	Convexity and reflexivity . . . . .	91
5.5	Coarse embeddings and finite subsets . . . . .	96
5.6	Expanders . . . . .	98
5.7	A geometric characterization of non-embeddability . . . . .	101
5.8	Compression of coarse embeddings . . . . .	108
5.9	Compression $> \frac{1}{2}$ implies property A . . . . .	110
	Exercises . . . . .	116
	Notes and remarks . . . . .	117
<b>6</b>	<b>Group actions on Banach spaces</b>	119
6.1	Affine isometric actions . . . . .	119
6.2	Metrically proper actions and a-T-menability . . . . .	121
6.3	Actions on $\ell_p$ -spaces and reflexive Banach spaces . . . . .	125
6.4	Kazhdan's property (T) . . . . .	128
6.5	Fixed points and Kazhdan's property (T) . . . . .	130
6.6	Construction of expanders . . . . .	132
6.7	Property (T) and spectral conditions . . . . .	134
	Exercises . . . . .	140
	Notes and remarks . . . . .	141
<b>7</b>	<b>Coarse homology</b>	143
7.1	Coarse locally finite homology . . . . .	143
7.2	Uniformly finite homology . . . . .	144
7.3	Eilenberg swindles and Ponzi schemes . . . . .	149
7.4	Aperiodic tiles and non-amenable spaces . . . . .	154
7.5	Coarsening homology theories . . . . .	160
7.6	The coarsening homomorphism . . . . .	162
	Exercises . . . . .	165
	Notes and remarks . . . . .	165

<b>8 Survey of applications</b>	<b>166</b>
8.1 Topological rigidity . . . . .	166
8.2 Geometric rigidity . . . . .	167
8.3 Index theory . . . . .	169
References	172
Index	187