## Notation and conventions

Below we list the notation and conventions used in this book. Most of them are standard.

## Sets

$\mathbb{N} \quad$ the set of natural numbers $(0 \notin \mathbb{N})$
$\mathbb{Z} \quad$ the set of integer numbers
$\mathbb{R} \quad$ the set of real numbers
$\mathbb{C} \quad$ the set of complex numbers
$\left(x_{i}\right)_{i \in I} \quad$ a sequence indexed by $I$
$\coprod_{i \in I} X_{i} \quad$ for a collection of sets $\left\{X_{i} \mid i \in I\right\}$, the disjoint sum of the sets $X_{i}$
$A \triangle B \quad$ symmetric difference of $A$ and $B$, i.e., $A \triangle B=(A \backslash B) \cup$ ( $B \backslash A$ )
$\operatorname{supp} f \quad$ the support of a function $f: X \rightarrow \mathbb{R}$, i.e., the set of those $x \in X$ for which $f(x) \neq 0$
$\operatorname{nbhd}_{R}(A) \quad$ the $R$-neighborhood of a subset $A \subseteq X$ of a metric space $X$, $\operatorname{nbhd}_{R}(A)=\{x \in X \mid d(x, A) \leq R\}$
$\partial_{r} A \quad$ the $r$-boundary of a subset $A \subseteq X$ of a metric space $X$, defined as $\{x \in X \backslash A \mid d(x, A) \leq r\}$
$\partial^{e} A \quad$ the edge boundary of a set $A \subseteq V$ in a graph $X=(V, E)$, defined as the set of those edges $e \in E$, which have exactly one vertex in $A$

## Numbers

```
# A
|x
|g
x\rfloor=\operatorname{sup}{z\in\mathbb{Z}|x\geqz}
(x|y)
\langlev,w\rangle
the number of elements of a set \(A\) the absolute value of \(x \in \mathbb{R}\) in a group \(G\), the length of an element \(g \in G\) the floor function of \(x \in \mathbb{R}\) the Gromov product of \(x\) and \(y\) with respect to \(p\) duality pairing of vectors
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## Groups

$e \in G \quad$ the identity element in the group $G$
$\mathbb{F}_{n} \quad$ the free group on $n$ generators
$g \cdot f \quad$ the translation action of $g \in G$ on a function $f: G \rightarrow X$,
defined by $(g \cdot f)(h)=f(h g)$

## Spaces

$$
\begin{aligned}
& B_{X}(x, r) \text { or } B(x, r) \quad \text { the closed ball of radius } r \text { centered at } x \in \\
& X, B_{X}(x, R)=\{y \in X \mid d(x, y) \leq R\} \\
& S_{X}(x, r) \text { or } S(x, R) \\
& f(X) \\
& \ell_{p}(I)=\left\{\left.\left(x_{i}\right)_{i \in I}\left|\sum_{i \in I}\right| x_{i}\right|^{p}<\infty\right\} \\
& \ell_{p}(I)_{1,+} \\
& \text { the sphere of radius } r \text { centered at } x \in X \text {, } \\
& S_{X}(x, R)=\{y \in X \mid d(x, y)=R\} \\
& \text { the image of the set } X \text { under a map } f \\
& \text { the } \ell_{p} \text {-space on } I \\
& \text { the set }\{f: I \rightarrow \mathbb{R} \mid f(i) \geq 0 \text { for all } \\
& \left.i \in I \text { and }\|f\|_{p}=1\right\} \\
& \left(\bigoplus X_{n}\right)_{(p)} \\
& \text { the direct sum of Banach spaces } X_{n} \text { with } \\
& \text { the norm }\left\|\left(x_{n}\right)\right\|_{p}=\left(\sum_{i=1}^{\infty}\left\|x_{n}\right\|^{p}\right)^{1 / p}
\end{aligned}
$$

## Functions

$1_{A}$ the characteristic function of a set $A$
$1_{x}$ the Dirac function at $x$

