Preface

In this book dynamical systems are investigated from a geometric viewpoint. A strong geometric property of a dynamical system is admitting an invariant manifold. In this case essential dynamics of the system takes place on some lower dimensional surface. Invariant manifolds are an important tool in a broad range of applications such as mechanical systems, chemical reaction dynamics, fluid mechanics, electronic circuit theory and singular perturbation theory. The theory of invariant manifolds for dynamical systems is well established. It goes back to the work of Hadamard [47] and Perron [105] and it was further developed by many authors. We mention Fenichel [38], [39], [40], Hirsch, Pugh, Shub [55], Kelley [63], Carr [23], Wiggins [129], Chaperon [24], [25]. The aim of this book is to present rigorous results on invariant manifolds in dynamical systems and to give examples of possible applications. The book is targeted at researchers in the field of dynamical systems interested in precise theorems easy to apply. Part III might also serve as an underlying text for a student seminar in mathematics.

Our approach to invariant manifolds for discrete dynamical systems is based on the so-called graph transform, already used by Hadamard [47]. Assume that the dynamical system is given by a map $P: X \times Y \to X \times Y$ where X, Y are nonempty open sets in some Banach spaces. We consider manifolds described as the graph M of a function $\sigma: X \to Y$. Under certain conditions the image of M under the map P is again a graph of a function $\bar{\sigma}: X \to Y$. This induces an operator $\mathcal{F}$ in the space of functions considered. An invariant manifold is obtained as a fixed point of the operator $\mathcal{F}$. The existence of a fixed point is established by the contraction principle.

We also consider continuous dynamical systems given by an ordinary differential equation (ODE). The time-T map of an ODE is a discrete dynamical system. The graph transform approach for maps carries over to ODEs, if applied to the time-T map. We give conditions on the vector field implying the existence of an invariant manifold for the ODE. Invariant manifold results for ODEs can also be derived by different approaches without using the graph transform. We mention Perron [105], Kelley [63], Knobloch, Kappel [72], Knobloch [69], Yi [130].

In the existing literature on invariant manifolds there is a strong tendency to formulate the results in a rather general setting. We aim at invariant manifold results in a simple setting, easy to apply and providing quantitative estimates. As in Kirchgraber, Lasagni, Nipp, Stoffer [66] we formulate conditions easy to verify and leading to sharp results if the coordinates are chosen in an appropriate way. In the discrete case we give conditions on the Lipschitz constants of the map and in the continuous case conditions on the derivatives of the vector field.

The book is organized as follows. In Part I discrete dynamical systems in Banach spaces are considered. We derive results on the existence of attractive and repulsive invariant manifolds that may be described as the graph of some function. We also treat manifolds described in several charts. In addition, we state results on the smoothness of the invariant manifold, on perturbations of the manifold and on the foliation of the adjacent space. In Part II we establish analogous results for continuous dynamical systems in finite dimensions. In Part III we apply the results of the first two parts. The emphasis is on applications to numerical analysis and to singularly perturbed systems of ODEs. In an appendix the hypotheses and conditions used in the theorems are arranged to help to navigate in the bulk of assumptions made.

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