

# Contents

Preface	v
<b>I Discrete Dynamical Systems – Maps</b>	<b>1</b>
<b>1 Existence</b>	<b>5</b>
1.1 Repulsive positively invariant manifolds . . . . .	5
1.2 Attractive negatively invariant manifolds . . . . .	13
1.3 Hyperbolic invariant manifolds . . . . .	20
1.4 Manifolds defined by several charts . . . . .	24
1.4.1 A general existence result . . . . .	24
1.4.2 Tools for Chapter 13 . . . . .	29
<b>2 Perturbation and approximation</b>	<b>33</b>
2.1 Attractive negatively invariant manifolds . . . . .	33
2.2 Repulsive positively invariant manifolds . . . . .	35
<b>3 Smoothness</b>	<b>37</b>
3.1 Repulsive positively invariant manifolds . . . . .	37
3.1.1 The first derivative . . . . .	38
3.1.2 The higher derivatives . . . . .	41
3.1.3 The differentiability with respect to parameters . . . . .	44
3.2 Attractive negatively invariant manifolds . . . . .	45
<b>4 Foliation</b>	<b>46</b>
4.1 The stable foliation . . . . .	46
4.2 The unstable foliation . . . . .	54
4.3 The hyperbolic case . . . . .	56
<b>5 Smoothness of the foliation with respect to the base point</b>	<b>59</b>
5.1 The stable foliation . . . . .	59
5.2 The unstable foliation . . . . .	68
<b>II Continuous Dynamical Systems – ODEs</b>	<b>69</b>
<b>6 A general result for the time-<math>T</math> map</b>	<b>71</b>
<b>7 Invariant manifold results</b>	<b>74</b>
7.1 Auxiliary results . . . . .	74
7.2 Attractive negatively invariant manifolds . . . . .	81
7.3 Repulsive positively invariant manifolds . . . . .	88

<b>III Applications</b>	93
<b>8 Fixed points and equilibria</b>	95
8.1 The local stable and unstable manifold of a hyperbolic fixed point . . .	95
8.2 The strongly stable manifold of an equilibrium . . . . .	99
<b>9 The one-step method associated to a linear multistep method</b>	101
9.1 Basic facts on linear multistep methods . . . . .	102
9.2 The associated one-step method . . . . .	103
9.3 The global error of linear multistep methods . . . . .	108
<b>10 Invariant manifolds for singularly perturbed ODEs</b>	110
10.1 Attractive manifolds . . . . .	110
10.2 Hyperbolic manifolds . . . . .	114
<b>11 Runge–Kutta methods applied to singularly perturbed ODEs</b>	121
11.1 Nonstiff methods . . . . .	123
11.2 Stiff methods . . . . .	126
11.2.1 The invariant manifold . . . . .	126
11.2.2 The global error . . . . .	133
<b>12 Invariant curves of perturbed harmonic oscillators</b>	137
12.1 The van der Pol equation . . . . .	137
12.1.1 The method of averaging for perturbed harmonic oscillators	139
12.1.2 The invariant manifold for perturbed harmonic oscillators . .	140
12.1.3 Application to the van der Pol equation . . . . .	142
12.2 The symplectic Euler method . . . . .	143
12.2.1 The method of averaging for the map . . . . .	146
12.2.2 The invariant manifold of the map . . . . .	148
12.2.3 Application to the van der Pol equation . . . . .	149
12.3 The Euler method . . . . .	152
<b>13 Blow-up in singular perturbations</b>	156
13.1 Introduction . . . . .	156
13.2 The main result . . . . .	157
13.3 Preliminaries . . . . .	159
13.3.1 The blow-up . . . . .	159
13.3.2 The reference manifold . . . . .	160
13.4 Proof of the main result . . . . .	161
13.4.1 The chart $\Phi_1$ . . . . .	162
13.4.2 The chart $\Phi_2$ . . . . .	165
13.4.3 The chart $\Phi_3$ . . . . .	170
13.4.4 The chart $\Phi_4$ . . . . .	174

13.4.5	The chart $\Phi_5$ . . . . .	178
13.4.6	The chart $\Phi_6$ . . . . .	182
<b>14</b>	<b>Application of Runge–Kutta methods to differential-algebraic equations</b>	<b>187</b>
<b>IV</b>	<b>Appendices</b>	<b>197</b>
<b>A</b>	<b>Hypotheses and conditions for maps</b>	<b>199</b>
A.1	Hypotheses . . . . .	199
A.2	Conditions . . . . .	202
<b>B</b>	<b>Hypotheses and conditions for ODEs</b>	<b>204</b>
B.1	Hypotheses . . . . .	204
B.2	Conditions . . . . .	205
	Bibliography	207
	Index	215