Preface

The purpose of this book is to present recent advances in the theory of local properties of mappings in the plane. The main emphasis is in the almost everywhere differentiable homeomorphic mappings. Thus quasiconformal and bi-Lipschitz mappings and the methods to study these mappings form the core of the book.

Quasiconformal mappings have turned out to be instrumental in the study of Riemann surfaces, Teichmüller spaces, Kleinian groups, meromorphic functions, holomorphic motion complex dynamics, Clifford analysis and tomography. Bi-Lipschitz mappings are used for example in the elasticity and control theory and there are unexpected connections between these mapping classes.

The book consists of three parts. The first part contains some problems from analysis and mathematical physics, the study of which leads naturally to the Beltrami equation and therefore to the quasiconformal mappings. This part is mainly introductory and intended to readers not familiar with quasiconformal mappings. Much of the material can be found in other books and hence several proofs are omitted. However, in Chapter 5.1 we present several examples of quasiconformal mappings which exhibit the complicated local behavior of quasiconformal mappings.

Part II is intended for researchers interested in new aspects of infinitesimal behavior of mappings. The compactness properties of quasiconformal mappings make it possible to study the infinitesimal behavior of a quasiconformal mapping at a point where the mapping is not differentiable. This leads to the concept of an infinitesimal space and the concept is used in subsequent chapters to study local properties of mappings. At the end of part II we consider classical local regularity results on the boundary from a new point of view.

In Part III we apply the quasiconformal function theory to study a non-linear elasticity problem and bi-Lipschitz mappings. New methods are used to study interior and boundary variation of quasiconformal mappings and criteria of univalence.

Throughout the book we have tried to illustrate the results by examples. Many of them have not been published in monographs before.

This book is addressed to the experts in modern geometric analysis, quasiconformal mappings and extensions, non-linear elasticity theory as well as to the beginning researchers and graduate students with a year's background in complex variables seeking access to research topics.

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