

Contents

Preface	v
I Quasiconformal Mappings in the Plane	1
1 Background of the theory	3
1.1 Introduction	3
1.2 Quasiconformal diffeomorphisms	6
1.3 Grötzsch's mapping problem	7
1.4 Conformal structures on surfaces	10
1.5 Conductivity in inhomogeneous media	12
1.6 Holomorphic motions	14
1.7 Streamlining of a ball in space	15
1.8 Bi-Lipschitz rotation	17
2 Conformal invariants	19
2.1 Introduction	19
2.2 Extremal length	19
2.3 Modulus of a curve family	21
2.4 Modulus of a circular annulus	25
2.5 Modulus of a rectangle	25
2.6 Modulus of quadrilaterals and ring domains	26
2.7 Grötzsch's and Teichmüller's modulus theorem	28
2.8 Diameter estimates	30
3 Definitions of quasiconformal maps	34
3.1 Introduction	34
3.2 Geometric definition	34
3.3 Absolute continuity on lines	35
3.4 Differentiability almost everywhere	35
3.5 Dilatation condition	36
3.6 Integrability condition	37
3.7 Pointwise dilatation	38
3.8 Astala's regularity theorem	39
3.9 Reich–Walczak's type integral modulus estimates	41
3.10 Analytic definition	46

4	Compactness and convergence theory	49
4.1	General convergence properties	49
4.2	Equicontinuity properties	53
4.3	Dilatation and convergence	56
5	Beltrami differential equation	58
5.1	Introduction	58
5.2	Cauchy and Hilbert transformations	59
5.3	Tricomi's type integral equations	62
5.4	Existence and representation theorems	65
5.5	Measurable Riemann Mapping Theorem	67
5.6	Higher integrability exponent	69
5.7	Dependence on parameter	70
5.8	Proof for Astala's theorem	74
5.9	Examples of quasiconformal mappings	77
5.10	Examples of quasiconformal mappings	77
II	Infinitesimal Geometry of Quasiconformal Maps	85
6	Infinitesimal space	87
6.1	Introduction	87
6.2	Convergence theorems and majorizing metrics	89
6.3	Definition of the infinitesimal space	91
6.4	Simple $T(z_0, f)$	92
6.5	Approximate continuity	98
6.6	Weak conformality property	99
6.7	Asymptotic symmetry	100
7	Asymptotically conformal curves	107
7.1	Introduction	107
7.2	Definition of asymptotically conformal curves	107
7.3	Asymptotic homogeneity and conformality	108
7.4	Criteria for asymptotic conformality	112
7.5	Asymptotic symmetry and conformality	113
8	Conformal differentiability	117
8.1	Introduction	117
8.2	Teichmüller–Wittich–Belinskiĭ's type theorem	119
8.3	Modulus estimates	120
8.4	Rotation theorems	121
8.5	Proof for Conformality Theorems	126
8.6	Conformality on a set	129

9	Points of maximal stretching	131
9.1	Introduction	131
9.2	Sufficient conditions	132
9.3	Maximal stretching criterion	135
10	Lipschitz continuity of quasiconformal maps	138
10.1	Introduction	138
10.2	Growth estimates	139
10.3	Conditions for Lipschitz continuity	141
10.4	Weak Lipschitz continuity	145
11	Regularity of quasiconformal curves	150
11.1	Introduction	150
11.2	Regularity of boundary correspondence	150
11.3	Smoothness of quasicircles	154
12	Regularity of conformal maps at the boundary	157
12.1	Introduction	157
12.2	Lindelöf’s smoothness theorem	157
12.3	Warschawski’s Conformality Theorem	159
III	Applications of Quasiconformal Maps	161
13	John’s rotation problem	163
13.1	Introduction	163
13.2	Main results	163
13.3	Proof of John’s rotation problem	165
13.4	Factoring of spiral-like maps	168
14	Variation of quasiconformal maps	172
14.1	Introduction	172
14.2	Variational procedure	172
14.3	Necessary conditions for extremum	173
14.4	Linear partial differential systems	175
15	Criteria of univalence	180
15.1	Introduction	180
15.2	Ahlfors–Weill’s criteria	181
	Bibliography	185
	Index	203