

## EMS Tracts in Mathematics 25

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# Metric Geometry of Locally Compact Groups

Winner of the 2016  
EMS Monograph Award



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# Foreword

This book offers to study locally compact groups from the point of view of appropriate metrics that can be defined on them, in other words to study “Infinite groups as geometric objects”, as Gromov writes it in the title of a famous article. The theme has often been restricted to finitely generated groups, but it can favourably be played for locally compact groups.

The development of the theory is illustrated by numerous examples, including matrix groups with entries in the the field of real or complex numbers, or other locally compact fields such as  $p$ -adic fields, isometry groups of various metric spaces, and, last but not least, discrete group themselves.

Word metrics for compactly generated groups play a major role. In the particular case of finitely generated groups, they were introduced by Dehn around 1910 in connection with the Word Problem.

Some of the results exposed concern general locally compact groups, such as criteria for the existence of compatible metrics (Birkhoff–Kakutani, Kakutani–Kodaira, Struble). Other results concern special classes of groups, for example those mapping onto  $\mathbf{Z}$  (the Bieri–Strebel splitting theorem, generalized to locally compact groups).

Prior to their applications to groups, the basic notions of coarse and large-scale geometry are developed in the general framework of metric spaces. Coarse geometry is that part of geometry concerning properties of metric spaces that can be formulated in terms of large distances only. In particular coarse connectedness, coarse simple connectedness, metric coarse equivalences, and quasi-isometries of metric spaces are given special attention.

The final chapters are devoted to the more restricted class of compactly presented groups, which generalize finitely presented groups to the locally compact setting. They can indeed be characterized as those compactly generated locally compact groups that are coarsely simply connected.