

Contents

<i>Prolégomènes francophones</i>	<i>vii</i>
<i>Preface</i>	<i>ix</i>
0 A magnetic story	1
0.1 A magnetic realm	1
0.2 A connection with waveguides	13
0.3 General presentation of the book	17
Part 1 Methods and examples	23
1 Elements of spectral theory	25
1.1 Spectrum	25
1.2 Min-max principle and spectral theorem	38
1.3 Simplicity and Harnack’s inequality	53
2 Examples	57
2.1 Harmonic oscillator	57
2.2 A δ -interaction	60
2.3 Robin Laplacians	63
2.4 De Gennes operator and applications	66
2.5 Analytic families	77
2.6 Examples of Feynman–Hellmann formulas	81
3 First semiclassical examples	87
3.1 Semiclassical estimate of the number of eigenvalues	87
3.2 Harmonic approximation in dimension one	91
3.3 Helffer–Kordyukov’s toy operator	94
4 From local models to global estimates	97
4.1 A localization formula	97
4.2 Agmon–Persson estimates	102
4.3 Applications	105

5	Birkhoff normal form in dimension one	117
5.1	Symplectic geometry and pseudo-differential calculus	117
5.2	Birkhoff normal form	122
Part 2	Main theorems	129
6	Spectral reductions	131
6.1	Vanishing magnetic fields and boundary	131
6.2	Magnetic Born–Oppenheimer approximation	136
6.3	Magnetic WKB expansions: examples	143
7	Magnetic wells in dimension two	149
7.1	Vanishing magnetic fields	149
7.2	Non-vanishing magnetic fields	152
8	Boundary magnetic wells in dimension three	161
8.1	Magnetic half-space	161
8.2	Magnetic wedge	166
8.3	Magnetic cone	170
9	Waveguides	177
9.1	Magnetic waveguides	177
9.2	Magnetic layers	187
9.3	Broken waveguides	189
10	On some connected non-linear problems	195
10.1	Non-linear magnetic eigenvalues	195
10.2	Non-linear dynamics in waveguides	197
Part 3	Spectral reductions	201
11	Electric Born–Oppenheimer approximation	203
11.1	Quasimodes	203
11.2	Essential spectrum and Agmon estimates	205
11.3	Projection argument	206
11.4	Accurate lower bound	209
11.5	An alternative point of view	212
12	Magnetic Born–Oppenheimer approximation	221
12.1	Quasimodes	221
12.2	Rough estimates of the eigenfunctions	224
12.3	Coherent states and microlocalization	226

13	Examples of magnetic WKB constructions	235
13.1	Vanishing magnetic fields	235
13.2	Curvature induced magnetic bound states	240
Part 4 Magnetic wells in dimension two		245
14	Vanishing magnetic fields in dimension two	247
14.1	Normal form	247
14.2	Agmon estimates	252
14.3	Projection argument	257
15	Non-vanishing magnetic fields	261
15.1	Magnetic Birkhoff normal form	261
15.2	Microlocalization	267
16	Semiclassical non-linear magnetic eigenvalues	273
16.1	About the concentration-compactness principle	273
16.2	Proof of the non-linear semiclassical asymptotics	280
Part 5 Boundary magnetic wells in dimension three		287
17	Magnetic half-space	289
17.1	Quasimodes	289
17.2	Agmon estimates	291
17.3	Relative polynomial localizations in the phase space	294
17.4	Localization induced by the effective harmonic oscillator	301
18	Magnetic wedge	305
18.1	Quasimodes	305
18.2	Agmon estimates	308
18.3	Projection method	310
19	Magnetic cone	313
19.1	Quasimodes in the axisymmetric case	313
19.2	Agmon estimates	316
19.3	Axisymmetry of the first eigenfunctions	318
19.4	Spectral gap in the axisymmetric case	320
19.5	Dimensional reduction for a general orientation	324
Part 6 Waveguides		327
20	Magnetic effects in curved waveguides	329
20.1	Two-dimensional waveguides	329

20.2	Three-dimensional waveguides	336
21	Spectrum of thin triangles and broken waveguides	343
21.1	Quasimodes and boundary layer	343
21.2	Agmon estimates and projection method	347
21.3	Reduction of the broken waveguide to the triangle	348
22	Non-linear dynamics in bidimensional waveguides	351
22.1	A priori estimates of the non-linearity	351
22.2	Lower bound of the energy and consequences	355
	Bibliography	361
	Index	379