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Augusto C. Ponce

# Elliptic PDEs, Measures and Capacities

From the Poisson Equation to  
Nonlinear Thomas–Fermi Problems



European Mathematical Society

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