



## EMS Series of Lectures in Mathematics

Edited by Ari Laptev (Imperial College, London, UK)

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Hans Triebel

# PDE Models for Chemotaxis and Hydrodynamics in Supercritical Function Spaces



European Mathematical Society

Author:

Hans Triebel  
Friedrich-Schiller-Universität Jena  
Fakultät für Mathematik und Informatik  
Institut für Mathematik  
07737 Jena  
Germany  
E-mail: [hans.triebel@uni-jena.de](mailto:hans.triebel@uni-jena.de)

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Contact address:  
European Mathematical Society Publishing House  
Seminar for Applied Mathematics  
ETH-Zentrum SEW A27  
CH-8092 Zürich  
Switzerland

Phone: +41 (0)44 632 34 36  
Email: [info@ems-ph.org](mailto:info@ems-ph.org)  
Homepage: [www.ems-ph.org](http://www.ems-ph.org)

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6. *Mathematical Treatment of the Axioms of Physics*. . . . To treat in the same manner [foundations of geometry], by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.

*The organic unity of mathematics is inherent in the nature of this science, for mathematics is the foundation of all exact knowledge of natural phenomena.*

(David Hilbert, Lecture delivered before the international congress of mathematicians at Paris in **1900**, [Hil02], [Rei70, Chapter X]).

*The purposes of the meeting are twofold:*

1. *To exhibit the vitality of mathematical research and to indicate some of its potential major growing points: these include some of the major classical problems (the Riemann Hypothesis, the Poincaré Conjecture, the regularity of three-dimensional fluid flows) as well as some of the recently developed major research programs like those associated with the names of Langlands and Thurston.*

2. *To point up the growing connections between the frontiers of research in the mathematical sciences and cutting-edge developments in such areas as physics, biology, computational science, computer science, and finance.*

(Felix E. Browder, president of the AMS, announcing the meeting ‘Mathematical Challenges of the 21st Century’, Univ. California, Los Angeles, **2000**, [Bro00]).