Introduction

The main topic of this book is the representation theory of finite dimensional associative algebras with an identity over a field, which currently can be regarded as the study of the categories of their finite dimensional modules and the associated combinatorial and homological invariants. A prominent role in the representation theory of finite dimensional algebras over fields is played by Frobenius algebras. The Frobenius algebras originated in the 1903 papers by Frobenius and received modern characterizations in a series of papers by Brauer, Nesbitt and Nakayama from 1937–1941. In particular, we may say that a finite dimensional algebra A over a field K is a Frobenius algebra if there exists a nondegenerate K-bilinear form $(-,-)\colon A \times A \to K$ which is associative, in the sense that $(ab,c) = (a,bc)$ for all elements a,b,c of A. Frobenius algebras are selfinjective algebras (projective and injective modules coincide), and the module category of every finite dimensional selfinjective algebra over a field is equivalent to the module category of a Frobenius algebra.

The book is divided into three volumes and its main aim is to provide a comprehensive introduction to the modern representation theory of finite dimensional algebras over fields, with special attention devoted to the representation theory of Frobenius algebras, or more generally selfinjective algebras. The book is primarily addressed to graduate students starting research in the representation theory of algebras, as well as to mathematicians working in other related fields. It is hoped that the book will provide a friendly access to the representation theory of finite dimensional algebras, as the only prerequisite is a basic knowledge of linear algebra. We present complete proofs of all results stated in the book. Moreover, a rich supply of examples and exercises will help the reader understand and master the theory presented in the book.

In the first volume of the book, "Frobenius Algebras I. Basic Representation Theory" [SY2], divided into six chapters, we provided a general introduction to basic results and techniques of the modern representation theory of finite dimensional algebras over fields, including the Morita equivalences and the Morita–Azumaya dualities for the module categories, and the Auslander–Reiten theory of irreducible homomorphisms and almost split sequences. The heart of the first volume is devoted to presenting fundamental classical as well as recent results concerning the selfinjective algebras and their module categories. Moreover, two chapters of the first volume are devoted to basic properties of two classical classes of Frobenius algebras formed by the Hecke algebras of finite Coxeter groups and the finite dimensional Hopf algebras.

In the second volume of the book we continue to present basic results and techniques of the modern representation theory of finite dimensional algebras over

fields as well as exhibit a new wide class of selfinjective algebras and describe their representation theory.

The second volume of the book is divided into four chapters, each of which is subdivided into sections. We start with Chapter VII presenting a rather detailed representation theory of finite dimensional hereditary algebras over fields. The indecomposable finite dimensional hereditary algebras over a field are divided into three disjoint subclasses (hereditary algebras of Dynkin type, of Euclidean type, and of wild type) according to the behaviour of the associated Euler quadratic form on their Grothendieck group. Chapter VIII is devoted to introducing the tilting theory of finite dimensional algebras over fields and describing basic properties of the tilted algebras, which are the endomorphism algebras of tilting modules over finite dimensional hereditary algebras. In Chapter IX we introduce a functorial approach to the representation theory of finite dimensional algebras over fields and elements of the theory of degrees of irreducible homomorphisms in module categories. As an application, we obtain a complete description of the shapes of infinite stable components in the Auslander–Reiten quivers of finite dimensional selfinjective algebras over fields. The final Chapter X is devoted to the theory of Hochschild extensions of finite dimensional algebras over fields by duality bimodules, which form a prominent wide class of finite dimensional selfinjective algebras. In particular, we describe completely the representation theory of arbitrary Hochschild extensions of finite dimensional hereditary algebras over fields by means of duality bimodules.

The third volume of the book, "Frobenius Algebras III. Orbit Algebras", will be devoted to the study of Frobenius algebras as the orbit algebras of repetitive categories of finite dimensional algebras over fields with respect to actions of admissible automorphism groups. In particular, we will introduce the covering techniques which frequently allow to reduce the representation theory of Frobenius algebras to the representation theory of algebras of small homological dimension. A prominent role in these investigations will be played by the tilting theory and the authors' theory of selfinjective algebras with deforming ideals.