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Shape Variation and Optimization

A Geometrical Analysis

European Mathematical Society

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Foreword

This book is essentially the English version of the one published in French by the same authors [188], except for some additions and updates. It originated from graduate courses given in past years at the Universities of Nancy, Besançon, and Rennes in France.

The goal of these graduate courses was to provide an introduction to modern approaches to shape optimization, relying only on undergraduate level as a prerequisite, but reaching actual, current open questions of this very active domain.

The book is written in this same initial spirit. Some specific directions are more particularly developed, but all necessary mathematical tools and proofs are then provided in order to offer a self-contained presentation.

For instance, we provide all necessary knowledge on the classical capacity associated with the energy space H^1. We also develop the particular case of shape optimization associated with the Dirichlet problem for the Laplace operator: this is a simple but typical example that is significant among all the main questions that arise in shape optimization associated with more complex systems of partial differential equations. As apparent from this model example, it is important to understand the behavior of these systems under variations of their underlying domains. This explains why the two keywords "variation" and "optimization" appear in the title of this book.

In the same spirit, we chose to devote one full and extended chapter to the main question of differentiation with respect to shapes. This is a rather difficult but unavoidable topic that can rapidly become technical. We aimed at a mathematically rigorous presentation while being at the same time concerned with providing efficient tools for the actual computations of the shape derivatives (rigorous analysis and efficient calculus are somehow antagonistic in this topic).

We have also described all the various topologies on open subsets of $\mathbb{R}^N$, which are mostly used in the variation of shapes and in continuity properties for the associated PDEs. We tried to provide some kind of "FAQ" on this question.

The last two chapters address different questions. One is about qualitative geometric properties of optimal shapes: We chose to present several explicit examples in order to describe as many different methods as possible. The other one contains an introduction to quite different points of view in shape optimization which are recent and still in progress.

And we thought it was interesting to add a bibliographical footnote each time a new (noncontemporary) mathematician was quoted. Among other sources, we used the excellent book [168] by B. Hauchecorne and D. Suratteau, as well as the rich website http://www-history.mcs.st-and.ac.uk/Indexes/Full_Alph.html.

We would like to thank all the colleagues who helped us with their remarks on the French version and on preliminary versions of this one, in particular Marc Dambrine,

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