# Introduction

Spectral graph theory starts by associating matrices to graphs — notably, the adjacency matrix and the Laplacian matrix. The general theme is then, firstly, to compute or estimate the eigenvalues of such matrices, and secondly, to relate the eigenvalues to structural properties of graphs. As it turns out, the spectral perspective is a powerful tool. Some of its loveliest applications concern facts that are, in principle, purely graph theoretic or combinatorial. To give just one example, spectral ideas are a key ingredient in the proof of the so-called friendship theorem: if, in a group of people, any two persons have exactly one common friend, then there is a person who is everybody's friend.

This text is an introduction to spectral graph theory, but it could also be seen as an invitation to algebraic graph theory. On the one hand, there is, of course, the linear algebra that underlies the spectral ideas in graph theory. On the other hand, most of our examples are graphs of algebraic origin. The two recurring sources are Cayley graphs of groups, and graphs built out of finite fields. In the study of such graphs, some further algebraic ingredients — for example, characters — naturally come up.

The table of contents gives, as it should, a good glimpse of where this text is going. Very broadly, the first half is devoted to graphs, finite fields, and how they come together. This part is meant as an appealing and meaningful motivation. It provides a context that frames and fuels much of the second, spectral, half.

Many sections have one or two exercises. They are optional, in the sense that virtually nothing in the main body depends on them. But the exercises are usually of the non-trivial variety, and they should enhance the text in an interesting way. The hope is that the reader will enjoy them. At any rate, solutions are provided at the end of the text.

We assume a basic familiarity with linear algebra, finite fields, and groups, but not necessarily with graph theory. This, again, betrays our algebraic perspective.

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