

## Frequently used notation

$\log :$	unless otherwise specified, $\log z$ denotes the principal determination of the logarithm of the non-zero complex number $z$ , that is, writing $z = re^{i\theta}$ with $r$ positive and $\theta$ in $(-\pi, \pi]$ , we have $\log z = \log r + i\theta$ .
$e :$	base of the natural logarithm.
$\deg :$	degree (of a polynomial, of an algebraic number).
$\det :$	determinant.
positive :	strictly positive.
$\lfloor x \rfloor :$	largest integer $\leq x$ .
$\lceil x \rceil :$	smallest integer $\geq x$ .
$\{ \cdot \} :$	fractional part.
$\  \cdot \  :$	distance to the nearest integer.
Card :	cardinality (of a finite set).
perfect power :	integer of the form $a^b$ , with $a \geq 1$ and $b \geq 2$ integers.
$p_1 < p_2 < \dots :$	the set of all prime numbers ranged in increasing order.
$q_1 < \dots < q_s :$	a collection of $s$ distinct prime numbers, not necessarily consecutive.
$P[n] :$	greatest prime factor of the integer $n$ , with $P[0] = P[\pm 1] = 1$ .
$\omega(n) :$	number of distinct prime factors of the positive integer $n$ , with $\omega(1) = 0$ .
$\varphi :$	Euler totient function (Definition D.3).
$\mu :$	Möbius function (Definition D.3).
$\Phi_d(X, Y) :$	homogeneous $d$ -th cyclotomic polynomial.
$h(\alpha) :$	logarithmic Weil height of the algebraic number $\alpha$ (Definition B.4).
$L(P) :$	length of the polynomial $P(X_1, \dots, X_n)$ (Definition B.9).
$ \cdot _p :$	$p$ -adic absolute value, normalized such that $ p _p = p^{-1}$ .
$v_p :$	$p$ -adic valuation, normalized such that $v_p(p) = 1$ .
$S :$	finite, non-empty set of prime numbers (or, sometimes, of places of an algebraic number field).

$[n]_S$ :	$S$ -part of the non-zero integer $n$ , defined by $[n]_S = \prod_{p \in S}  n _p^{-1}$ .
$S$ -unit :	rational number whose numerator and denominator are only composed of prime numbers in $S$ .
integral $S$ -unit :	rational integer being an $S$ -unit.
$a \ll b$ :	the quantity $a$ is less than $b$ times an absolute, positive, effectively computable real number.
$a \ll_{c_1, \dots, c_n} b$ :	the quantity $a$ is less than $b$ times an effectively computable, positive, real number, which depends at most on $c_1, \dots, c_n$ .
$a \ll^{\text{ineff}} b$ :	the quantity $a$ is less than $b$ times an absolute, positive real number.
$\log_p, \exp_p$ :	$p$ -adic logarithm and $p$ -adic exponential functions.
$\mathcal{S}_N$ :	set of permutations of $\{1, \dots, N\}$ .
$e_K, f_K$ :	ramification index and residue degree of a $p$ -adic field $K$ .
$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{p}$ :	ideals of an algebraic number field.
$e_{\mathfrak{p}}, f_{\mathfrak{p}}$ :	ramification index and residue degree of an ideal $\mathfrak{p}$ .
$\cdot^\sigma$ :	Galois conjugacy.
$K, O_K, O_K^*, M_K, M_K^\infty, D_K, h_K$ :	an algebraic number field, its ring of integers, group of units, set of places, set of infinite places, discriminant, and class number.