

Prologue

We can straight away give the most general “definition” of a Kac–Moody group:

A **Kac–Moody group** is a group object associated to a Kac–Moody algebra.

This might seem like stating the obvious, but it reflects the fact that there are several natural ways to associate to a given Kac–Moody algebra $\mathfrak{g}(A)$ a group object, and that this yields in general several non-isomorphic objects, once one has made up one’s mind about the category in which a Kac–Moody group should live (and again, the choice of this category is not canonical).

The purpose of this third part is to introduce the known constructions of Kac–Moody groups, and to show how these can be related to one another. We will in particular distinguish two classes of Kac–Moody groups: the *minimal Kac–Moody groups*, which are obtained by “exponentiating” the *real* root spaces of the Kac–Moody algebra $\mathfrak{g}(A)$ (i.e. those associated with a real root), and the *maximal Kac–Moody groups*, in which all root spaces (*real* and *imaginary*) of $\mathfrak{g}(A)$ are taken into account.