Preface to the German Edition

Inequalities for differential operators of the kind considered in this book play a fundamental role in the modern theory of partial differential equations. Among the numerous applications of such inequalities are existence and uniqueness theorems, error estimates for numerical approximation of solutions and for residual terms in asymptotic formulas, as well as results on the structure of the spectrum. The inequalities arise in a wide range of topics and differ by the choice of differential operators and boundary conditions, by requirements on the boundaries of domains, and by the norms in the relevant function spaces.

For general differential operators with constant coefficients considered in this book, estimates in L^2 for functions with compact support in a domain have been extensively studied in [H55].

Estimates up to the boundary are much less studied. Estimates of such type can be found in the papers of Aronszajn [Aro54], Agmon [Agm58] (coercivity of differential operators and integro-differential forms), Schechter [Sch63], [Sch64], [Sch64a] (sufficient conditions for dominance in a half-space) and in other publications that will be discussed in the bibliographical notes at the end of each chapter.

The subject of this book is estimates for differential operators with constant coefficients in a half-space. There are no a priori restrictions on the type of considered differential operators.

The right-hand sides of the studied integral inequalities involve matrix differential operators or scalar differential operators in a half-space as well as boundary operators. Conditions under which the above-mentioned system of operators "dominates" an individual differential operator in a half-space or on its boundary are completely described. Applications of these results to the theory of well-posed boundary value problems in a half-space are given.

The domains of the relevant maximal operators are investigated in detail. In particular, the maximal operators weaker than the given one are described and a complete characterization of boundary values of functions from the specified domain is presented.

The results are complete. To a large extent, they are necessary and sufficient conditions. From these, more evident sufficient conditions are derived. General criteria are systematically applied to certain types of operators, in particular, to classical equations and systems of mathematical physics (Lamé's system of static elasticity theory, the linearized Navier–Stokes system, Cauchy–Riemann operators, Schrödinger operators, and so on).

The known results of Aronszajn, Agmon–Douglis–Nirenberg, Schechter fall into the general scheme and are sometimes strengthened.

This monograph does not overlap with the content of other books on linear differential operators and results presented have so far only been published in journal papers. The book summarizes the joint work of the authors on this topic during the period 1972–1977.

The authors hope that the book will be interesting and useful to a wide audience. It is intended for specialists and graduate students specializing in the theory of differential equations.

The reader is expected to be familiar with elements of the theory of ordinary differential equations, functional analysis, the theory of partial differential equations, and basics of linear algebra.

The content of the book is detailed in the introductions to each of its four chapters.

The authors thank the translator and the publisher for high quality of translation.

I. V. Gel′man
V. G. Maz′ya